Two-Loop Soft Corrections to QCD Dipole Showers

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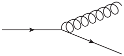
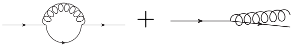
Radiative corrections as a branching process

[Marchesini,Webber] NPB238(1984)1, [Sjöstrand] PLB157(1985)321

- Make two well motivated assumptions
  - Parton branching can occur in two ways



\- observed



\- unobserved

  - Evolution conserves probability
- The consequence is Poisson statistics
  - Let the decay probability be λ
  - Assume indistinguishable particles $\rightarrow$ naive probability for n emissions

$$P_{\text{naive}}(n,\lambda) = \frac{\lambda^n}{n!}$$

  - Probability conservation (i.e. unitarity) implies a no-emission probability

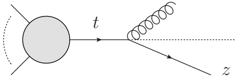
$$P(n,\lambda) = \frac{\lambda^n}{n!}\exp\{-\lambda\} \qquad \longrightarrow \qquad \sum_{n=0}^{\infty} P(n,\lambda) = 1$$

- In the context of parton showers $\Delta = \exp\{-\lambda\}$ is called Sudakov factor

Radiative corrections as a branching process

- Decay probability for parton state in collinear limit

$$\lambda \to \frac{1}{\sigma_n}\int_t^{Q^2} \mathrm{d}\bar{t}\,\frac{\mathrm{d}\sigma_{n+1}}{\mathrm{d}\bar{t}} \approx \sum_{\text{jets}} \int_t^{Q^2} \frac{\mathrm{d}\bar{t}}{\bar{t}} \int \mathrm{d}z\,\frac{\alpha_s}{2\pi} P(z)$$



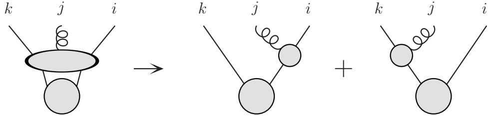
Parameter t identified with evolution "time"

- Soft double counting problem [Marchesini,Webber] NPB310(1988)461
  full soft radiation probability in all collinear regions

$$\frac{1}{t}\,\frac{2\,z}{1-z} \longrightarrow \frac{p_i p_k}{(p_i q)(q p_k)}$$

- Can be solved for single emission by partial fractioning
  and matching to collinear sectors [Catani,Seymour] hep-ph/9605323

$$\frac{p_i p_k}{(p_i p_j)(p_j p_k)} \to \frac{1}{p_i p_j}\,\frac{p_i p_k}{(p_i + p_k) p_j} + \frac{1}{p_k p_j}\,\frac{p_i p_k}{(p_i + p_k) p_j}$$



Color coherence and the dipole picture

- Splitting kernels become dependent on anti-collinear direction usually defined by color spectator in large-N_c limit
- Singularity confined to soft-collinear region only captures soft coherence effects at leading color

$$\frac{1}{1-z} \to \frac{1-z}{(1-z)^2+\kappa^2} \qquad \kappa^2 = \frac{k_\perp^2}{Q^2}$$

- Complete set of leading-order splitting functions now given by

$$P_{qq}(z,\kappa^2) = C_F\left[\frac{2\,(1-z)}{(1-z)^2+\kappa^2} - (1+z)\right]$$

$$P_{qg}(z,\kappa^2) = C_F\left[\frac{1+(1-z)^2}{z}\right], \qquad P_{gq}(z,\kappa^2) = T_R\Big[z^2+(1-z)^2\Big]$$

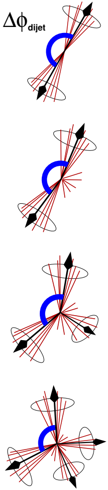
$$P_{gg}(z,\kappa^2) = 2\,C_A\left[\frac{1-z}{(1-z)^2+\kappa^2} + \frac{1}{z} - 2 + z(1-z)\right]$$

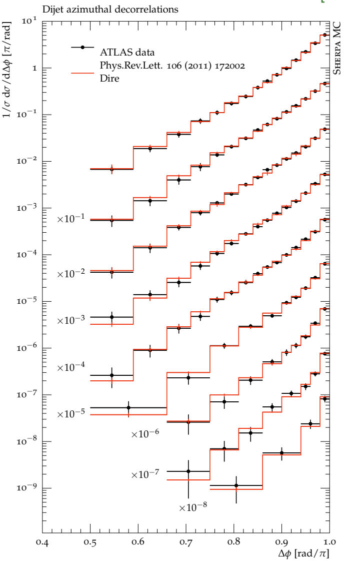
- Close correspondence to principal value regularization
[Curci,Furmanski,Petronzio] NPB175(1980)27

Accuracy and precision

Accuracy of the parton shower: Jets at the LHC

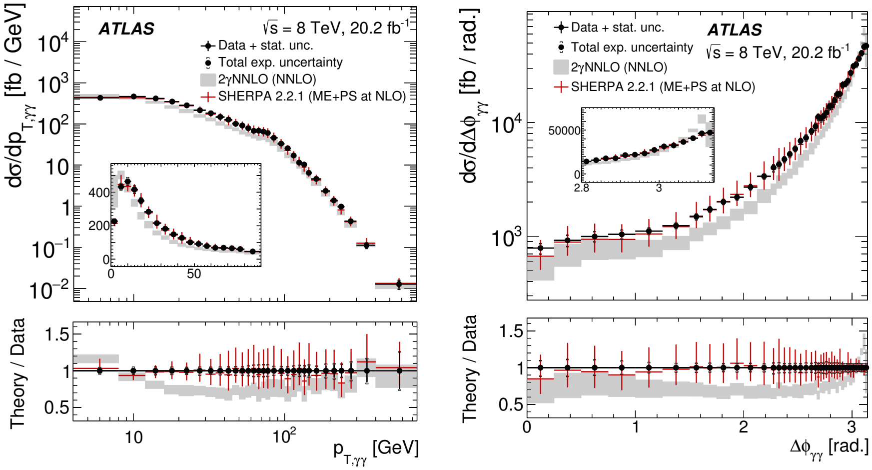
[Prestel,SH] arXiv:1506.05057





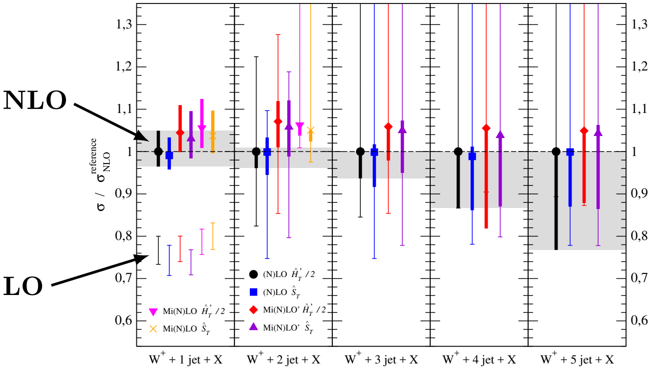
Accuracy of the parton shower: Photons at the LHC

[ATLAS] arXiv:1704.03839



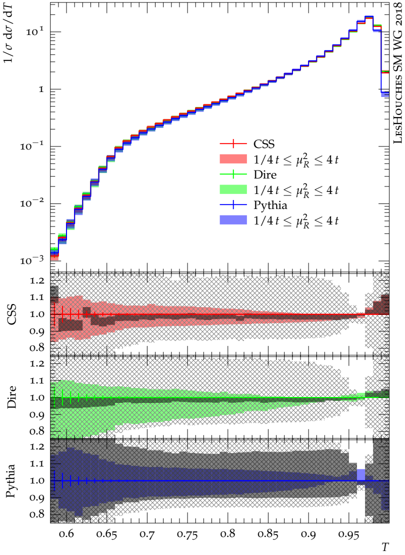
[Bern et al.] arXiv:1304.1253, arXiv:1412.4775
[Anger,Febres Cordero,Maître,SH] arXiv:1712.08621

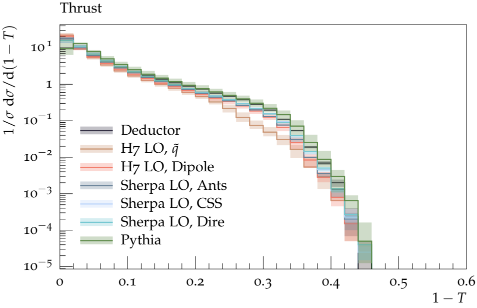
- $W^\pm$+jets at 13 TeV LHC, computed with BlackHat+Sherpa
- Largely reduced uncertainties at NLO, but more importantly good agreement for different functional forms of scale, including several variants of MINLO [Hamilton,Nason,Zanderighi] arXiv:1206.3572



Precision of parton-showers?

[LesHouches] arXiv:1605.04692, arXiv:1803.07977





- Assessment of PS uncertainties assuming they can be covered by varying evolution variable t in 2^{nd} order soft emission term

$$\frac{1}{t}\left(\frac{\alpha_s(t)}{2\pi}\right)^2\left[\beta_0(t)\log\frac{k_T^2}{t}+K(t)\right]\frac{2}{1-z}$$

Lessons learned and motivation of this talk

- Fueled by the NLO revolution, large parts of the MC community worked on precision fixed-order calculations during the past decade(s)
- This lead to much improved agreement with data and tremendous new capabilities of event generators $\rightarrow$ parton showers to catch up
- Many of the challenges at higher luminosity / energy require increased precision in the parton-shower simulation (we cannot hope to compute, say, 8-jet final states at NLO)

- Must understand what precision means in context of parton showers and how momentum conservation & unitarity impede comparison to analytical results at any given logarithmic order
- Here: Start to improve formal precision of parton shower by adding higher-order corrections to soft evolution
- There are many other questions, but we have to start somewhere
  We know that what's done here *must* feed into any full solution

How to make parton showers more precise?
NLO corrections to soft-gluon radiation

Soft evolution at the next-to-leading order

[Marchesini,Korchemsky] PLB313(1993)433, hep-ph/9210281

- Soft-gluon resummed expression of Drell-Yan or DIS cross section

$$\frac{1}{\sigma}\frac{d\sigma(z,Q^2)}{d\log Q^2} = \mathcal{H}(Q^2)\,\widetilde{W}(z,Q^2)$$

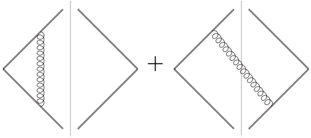
RGE governed by Wilson loop $\widetilde{W}$ ($Q(1-z)$ - total soft gluon energy)

- Non-abelian exponentiation theorem allows to expand as

$$\widetilde{W} = \exp\left\{\sum_{i=1}^{\infty} w^{(\mathrm{n})}\right\}$$

- One-loop result given by

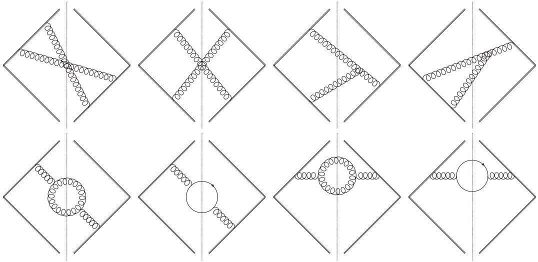
$$w^{(1)} = C_F\,\frac{\alpha_s(\mu)}{2\pi}\left[\ln^2 L + \frac{\pi^2}{6}\right] \quad \leftrightarrow$$



where $L = -b_+ b_- / b_0^2$ and $b_0 = 2\,e^{-\gamma_E}/\mu$

Soft evolution at the next-to-leading order

- 2-loop contribution $w^{(2)}$ computed from (reals only) [Belitsky] hep-ph/9808389



- Renormalized result in position space

$$w^{(2)} = C_F \frac{\alpha_s^2(\mu)}{(2\pi)^2} \left[-\frac{\beta_0}{6} \ln^3 L + \Gamma_{\text{cusp}}^{(2)} \ln^2 L + 2 \ln L \left(\Gamma_{\text{soft}}^{(2)} + \frac{\pi^2}{12} \beta_0 \right) + \ldots \right]$$

$$\Gamma_{\text{cusp}}^{(2)} = \left(\frac{67}{18} - \frac{\pi^2}{6} \right) C_A - \frac{10}{9} T_R n_f \,, \qquad \beta_0 = \frac{11}{6} C_A - \frac{2}{3} T_R n_f$$

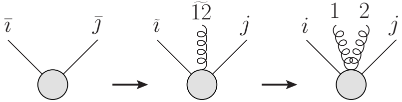
$$\Gamma_{\text{soft}}^{(2)} = \left(\frac{101}{27} - \frac{11}{72} \pi^2 - \frac{7}{2} \zeta_3 \right) C_A - \left(\frac{28}{27} - \frac{\pi^2}{18} \right) T_R n_f$$

- This is the benchmark to be reproduced by exclusive MC simulation

Separation of soft and collinear sectors

[Dulat,Prestel,SH] arXiv:1805.03757

- Phase space parametrized in terms of total soft momentum $q = p_1 + p_2$



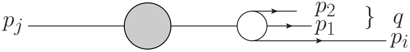
- Momentum space result expanded in Laurent series using

$$\frac{1}{q_\pm^{1+\varepsilon}} = -\frac{1}{\varepsilon}\,\delta(q_\pm) + \sum_{i=0}^{\infty} \frac{\varepsilon^n}{n!} \left(\frac{\ln^n q_\pm}{q_\pm}\right)_+$$

- Unitarity implies that factorized plus distributions like $[1/q_+]_+[1/q_-]_+$ have no PS analogue $\to$ define double-plus distributions instead

$$\big[f(q_+, q_-)\big]_{++}\, g(q_+, q_-) = f(q_+, q_-)\Big(g(q_+, q_-) - g(0,0)\Big)$$

- Re-organize entire calculation in terms of pure soft & collinear terms
  Key observation: $q_\pm = 0$ implies collinear limit both for 1 & 2 emissions



Soft evolution at the next-to-leading order

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[Catani,Grazzini] hep-ph/9908523

- Real-emission corrections can be written in convenient form

$$\mathcal{S}_{ij}^{(q\bar{q})}(1,2) = -\frac{s_{ij}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2})}\frac{T_R}{s_{12}}\Big(1-4\,z_1z_2\cos^2\phi_{12,ij}\Big)$$

$$\mathcal{S}_{ij}^{(gg)}(1,2) = \mathcal{S}_{ij}^{(\text{s.o.})}(1,2)\,\frac{C_A}{2}\left(1+\frac{s_{i1}s_{j1}+s_{i2}s_{j2}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2})}\right) + \frac{s_{ij}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2})}\frac{C_A}{s_{12}}\Big(-2+4\,(1-\varepsilon)\,z_1z_2\cos^2\phi_{12,ij}\Big)$$

- Strongly ordered and spin correlation components

$$\mathcal{S}_{ij}^{(\text{s.o.})}(1,2) = \frac{s_{ij}}{s_{i1}s_{12}s_{j2}} + \frac{s_{ij}}{s_{j1}s_{12}s_{i2}} - \frac{s_{ij}^2}{s_{i1}s_{j1}s_{i2}s_{j2}}$$

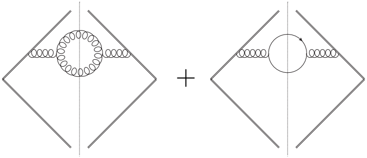
$$4\,z_1z_2\cos^2\phi_{12,ij} = \frac{(s_{i1}s_{j2}-s_{i2}s_{j1})^2}{s_{12}s_{ij}(s_{i1}+s_{i2})(s_{j1}+s_{j2})}$$

- Apparently simple structure, but unlike collinear NLO results not reflected by iterated leading-order splitting kernels
$\rightarrow$ not all denominators can be composed from LO expressions

NLO subtraction: Dipole approach

[Dulat,Prestel,SH] arXiv:1805.03757

- Nearly ok subtraction obtained from spin correlated parton shower



$$+ \ldots = \sum_{b=q,g} J_{ij,\mu}(p_{12}) J_{ij,\nu}(p_{12}) \frac{P^{\mu\nu}_{gb}(z_1)}{s_{12}}$$

- Building blocks are eikonal currents

$$J^\mu_{ij}(q) = \frac{p^\mu_i}{2p_i q} - \frac{p^\mu_j}{2p_j q}$$

and collinear splitting functions

$$P^{\mu\nu}_{gq}(z) = T_R \left(-g^{\mu\nu} + 4\, z(1-z)\, \frac{k^\mu_\perp k^\nu_\perp}{k^2_\perp} \right)$$

$$P^{\mu\nu}_{gg}(z) = C_A \left(-g^{\mu\nu} \left(\frac{z}{1-z} + \frac{1-z}{z} \right) - 2\,(1-\varepsilon) z(1-z)\, \frac{k^\mu_\perp k^\nu_\perp}{k^2_\perp} \right)$$

- Finite remainder has integrable singularities $\rightarrow$ not suitable for MC problem arises from interference of abelian & non-abelian diagrams

NLO subtraction: Antenna approach – Kinematics

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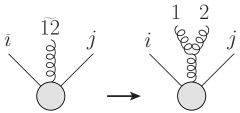
[Dulat,Prestel,SH] arXiv:1805.03757

- In iterated emission $\bar{\imath}\,\bar{\jmath} \to \widetilde{\imath}\,\widetilde{12}\,j \to ij12$ emission probability of first step written in terms of momenta after second step is

$$\frac{\tilde{p}_i p_j}{2(\tilde{p}_i\tilde{p}_{12})(\tilde{p}_{12}p_j)} = \frac{s_{ij}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2}) - s_{ij}s_{12}}$$



- Not identical to desired "eikonal" $s_{ij}/((s_{i1}+s_{i2})(s_{j1}+s_{j2}))$ in soft⊗collinear terms of $\mathcal{S}_{ij}$ but easily corrected by weight

$$w_{ij}^{12} = 1 - \frac{s_{ij}s_{12}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2})} = \left(\frac{p_{\perp,12}^{(ij)}}{m_{\perp,12}^{(ij)}}\right)^2$$

- Iterated eikonals of type $s_{ij}/(s_{i1}s_{j1})$, $s_{j1}/(s_{12}s_{j2})$ in $\mathcal{S}_{ij}^{(\mathrm{s.o.})}$ reconstructed by partial fractioning & matching to LO^2 $\to$ additional weight

$$\bar{w}_{ij}^{12} = \frac{(s_{i1}+s_{i2})(s_{j1}+s_{j2}) - s_{ij}s_{12}}{s_{i1}s_{j1}+s_{i2}s_{j2}} = \frac{\big(p_{\perp,12}^{(ij)}\big)^2}{\big(p_{\perp,1}^{(ij)}\big)^2 + \big(p_{\perp,2}^{(ij)}\big)^2}$$

- These weights lie between zero and one and reduce emission rates

Leading color fully differential soft evolution at NLO

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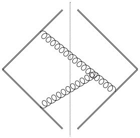
[Dulat,Prestel,SH] arXiv:1805.03757

- Squared LO eikonal and negative term in $\mathcal{S}_{ij}^{(\mathrm{s.o.})}$ both have no parton-shower analogue $\rightarrow$ correct for both mismatches by adding sub-leading color contribution to $i1$-collinear splitting functions

$$P_{ij,A}^{(\mathrm{slc})}(1,2) = \frac{2\,s_{ij}}{s_{i1}+s_{j1}}\,\frac{w_{ij}^{12}+\bar{w}_{ij}^{12}}{2}\left(\bar{C}_{ij}-C_A\right)\,, \quad \bar{C}_{ij} = \begin{cases} 2C_F & \text{if } i \text{ \& } j \text{ quarks} \\ C_A & \text{else} \end{cases}$$

- Second soft emission off Wilson lines occurs with color charge factor C_A due to interference with octet



$$P_{ij,B}^{(\mathrm{slc})}(1,2) = \frac{2\,s_{i2}}{s_{i1}+s_{12}}\,\frac{w_{ij}^{12}+\bar{w}_{ij}^{12}}{2}\left(C_A-\bar{C}_{ij}\right)$$

- Combined effect on $i1$-collinear matched splitting function

$$P_{ij}^{(\mathrm{slc})}(1,2) = \left(C_A-\bar{C}_{ij}\right)\left(\frac{2\,s_{i2}}{s_{i1}+s_{12}}-\frac{2\,s_{ij}}{s_{i1}+s_{j1}}\right)\frac{w_{ij}^{12}+\bar{w}_{ij}^{12}}{2}$$

- Non-singular in $i1$-collinear limit $\rightarrow$ color charges of Wilson lines in soft-collinear limit are C_i and C_j, in agreement with DGLAP

Leading color fully differential soft evolution at NLO

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[Dulat,Prestel,SH] arXiv:1805.03757

- Complete NLO-weighted LO splitting functions

$$(P_{qq})_i^k(1,2) = C_F\left(\frac{2\,s_{i2}}{s_{i1}+s_{12}}\,\frac{w_{ik}^{12}+\bar{w}_{ik}^{12}}{2}\right) + P_{ik}^{(\mathrm{slc})}(1,2)$$

$$(P_{gg})_{ij}(1,2) = C_A\left(\frac{2\,s_{i2}}{s_{i1}+s_{12}}\,\frac{w_{ij}^{12}+\bar{w}_{ij}^{12}}{2} + w_{ij}^{12}\left(-1+z(1-z)\,2\cos^2\phi_{12}^{ij}\right)\right)$$

$$(P_{gq})_{ij}(1,2) = T_R\,w_{ij}^{12}\left(1-4z(1-z)\cos^2\phi_{12}^{ij}\right)$$

- Calculation completed by subtracted real correction, virtuals and factorization counterterms
- Counterterms are endpoint contributions

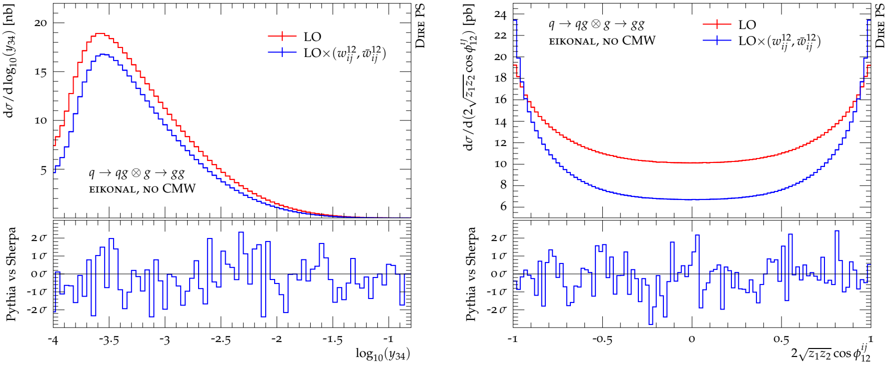
$$\tilde{\mathcal{S}}_{gq}^{(\mathrm{cusp})} = \delta(s_{12})\,\frac{2\,s_{ij}}{s_{i12}s_{j12}}\,T_R\Big[2z(1-z) + \big(1-2z(1-z)\big)\ln(z(1-z))\Big]$$

$$\tilde{\mathcal{S}}_{gg}^{(\mathrm{cusp})} = \delta(s_{12})\,\frac{2\,s_{ij}}{s_{i12}s_{j12}}\,2C_A\left[\frac{\ln z}{1-z} + \frac{\ln(1-z)}{z} + \big(-2+z(1-z)\big)\ln(z(1-z))\right]$$

$$\tilde{\mathcal{S}}_{wl}^{(\mathrm{cusp})} = -\,\delta(s_{i1})\,\frac{1}{2}\,\frac{C_A}{2}\,\frac{2\,s_{ij}}{s_{i12}s_{j12}}\left(\frac{\ln z_i}{1-z_i} + \frac{\ln(1-z_i)}{z_i}\right) + \big(\text{swaps}\big)$$

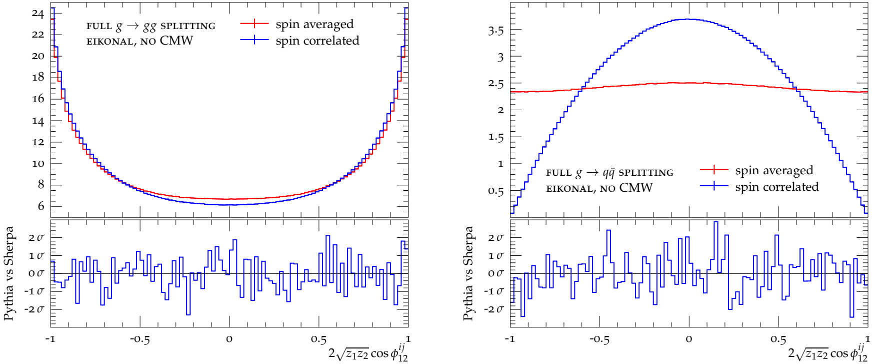
Sum integrates to CMW correction [Catani,Marchesini,Webber] NPB349(1991)635

Leading color fully differential soft evolution at NLO



- Impact of weights w_{ij}^{12} and $\bar{w}_{ij}^{12}$ on $3 \to 4$ Durham jet rate and azimuthal angle between soft gluons at LEP I

Leading color fully differential soft evolution at NLO



- Impact of spin correlations on azimuthal angle between soft gluons / quarks at LEP I

Leading color fully differential soft evolution at NLO

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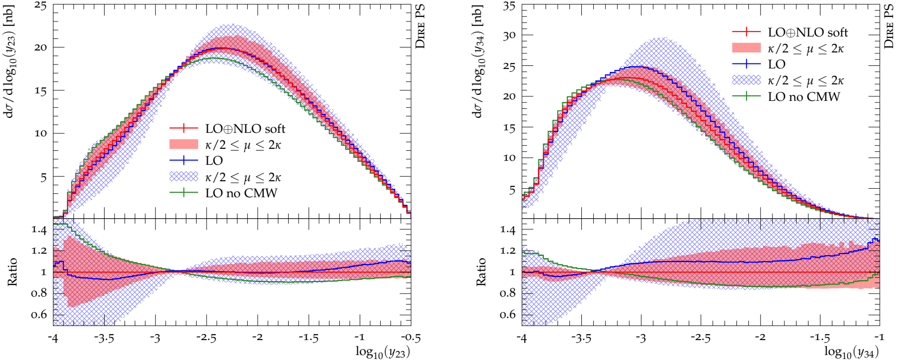
[Dulat,Prestel,SH] arXiv:1805.03757



- Impact on $2 \to 3$ and $3 \to 4$ Durham jet rate at LEP I
- Uncertainty bands no longer just estimates
  but perturbative QCD predictions for the first time
- Fair agreement with CMW scheme

Summary

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- Double-soft & flavor-changing triple-collinear NLO corrections now added to parton-shower simulation in fully exclusive form
- Remaining collinear NLO corrections to be added by subtracting double-soft components as needed and using CS-like subtraction devised in flavor-changing case
- Relation to NLL resummation to be investigated in detail
  $\rightarrow$ benchmark using [Dasgupta,Dreyer,Hamilton,Monni,Salam] arXiv:1805.09327

Thank you for your attention