Towards an NLO parton shower
and improved uncertainty estimates

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How can we make parton showers more precise?

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- Formulate parton-shower algorithm at NLO [Nagy,Soper] arXiv:1705.08093
  Naturally, NLO DGLAP evolution must be part of the full solution
- NLO DGLAP splitting kernels known since long
  [Curci,Furmanski,Petronzio] NPB175(1980)27, PLB97(1980)437
  [Floratos,Kounnas,Lacaze] NPB192(1981)417
- So far not implemented in parton showers because
  - Kernels are scheme dependent (easy)
  - Overlap with soft-gluon resummation (hard)
- Focus on purely collinear corrections ($\leftrightarrow$ B2) for a start
  - Redefine time-like Sudakovs to recover NLO DGLAP evolution
    [Jadach,Skrzypek] hep-ph/0312355
  - Negative NLO corrections require weighted veto algorithm
    [Schumann,Siegert,SH] arXiv:0912.3501, [Lönnblad] arXiv:1211.7204
  - Flavor changing splitting functions require $2 \to 4$ transitions
    [Prestel,SH] arXiv:1705.00742
- Flavor-changing case is simplest but requires all the technology

Parton-shower interpretation of the DGLAP equations

- DGLAP equation for fragmentation functions

$$\frac{\mathrm{d}x\,D_a(x,t)}{\mathrm{d}\ln t} = \sum_{b=q,g}\int_0^1 \mathrm{d}\tau\int_0^1 \mathrm{d}z\,\frac{\alpha_s}{2\pi}\big[zP_{ab}(z)\big]_+\,\tau D_b(\tau,t)\,\delta(x-\tau z)$$

- Define plus prescription $\big[zP_{ab}(z)\big]_+ = \lim\limits_{\varepsilon\to 0} zP_{ab}(z,\varepsilon)$

$$P_{ab}(z,\varepsilon) = P_{ab}(z)\,\Theta(1-z-\varepsilon) - \delta_{ab}\sum_{c\in\{q,g\}}\frac{\Theta(z-1+\varepsilon)}{\varepsilon}\int_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,P_{ac}(\zeta)$$

- Rewrite for finite ε

$$\frac{\mathrm{d}\ln D_a(x,t)}{\mathrm{d}\ln t} = -\sum_{c=q,g}\int\limits_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,\frac{\alpha_s}{2\pi}P_{ac}(\zeta) + \sum_{b=q,g}\int\limits_x^{1-\varepsilon}\frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)\,\frac{D_b(x/z,t)}{D_a(x,t)}$$

- First term is derivative of Sudakov factor

$$\Delta_a(t_0,t) = \exp\left\{-\int_{t_0}^{t}\frac{\mathrm{d}\bar{t}}{\bar{t}}\sum_{c=q,g}\int_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,\frac{\alpha_s}{2\pi}P_{ac}(\zeta)\right\}$$

Parton-shower interpretation of the DGLAP equations

- Use generating function $\mathcal{D}_a(x,t,\mu^2) = D_a(x,t)\Delta_a(t,\mu^2)$ to write

$$\frac{\mathrm{d}\ln\mathcal{D}_a(x,t,\mu^2)}{\mathrm{d}\ln t} = \sum_{b=q,g}\int_x^{1-\varepsilon}\frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)\,\frac{D_b(x/z,t)}{D_a(x,t)}\ .$$

- A similar probability density is used to generate initial-state emissions
  But final-state showers are typically unconstrained (hadrons not identified)
  In this case the probability density is modified to

$$\frac{\mathrm{d}}{\mathrm{d}\ln t}\ln\left(\frac{\mathcal{D}_a(x,t,\mu^2)}{D_a(x,t)}\right) = \sum_{b=q,g}\int_0^{1-\varepsilon}\mathrm{d}z\,z\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)\ .$$

- **Net result**: Unitarity implies that forward-branching Sudakovs must include a 'symmetry factor' z [Jadach,Skrzypek] hep-ph/0312355
- Convenient interpretation as "tagging" of evolving parton
- Equivalent to standard technique at LO due to symmetry of $P_{ab}(z)$
  More care is needed at NLO [Prestel,SH] arXiv:1705.00742

$2 \to 4$ kinematics mapping, massless FF case

- Define evolution & splitting variables

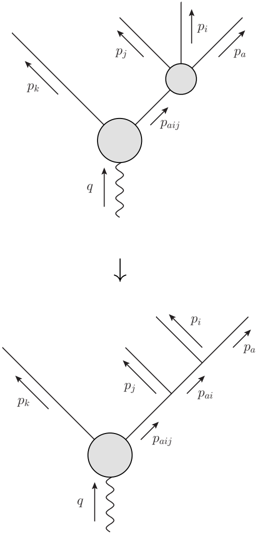
$$t = \frac{4\,p_j p_{ai}\,p_{ai} p_k}{q^2}\;,\qquad z_a = \frac{2\,p_a p_k}{q^2}$$
$$s_{ai} = 2\,p_a p_i\;,\qquad x_a = \frac{p_a p_k}{p_{ai} p_k}$$

- First branching $(\widetilde{aij}, \tilde{k}) \to (ai, j, k)$ constructed with $m_{ai}^2 \to s_{ai}$, using
  [Catani,Dittmaier,Seymour,Trocsanyi] hep-ph/0201036
  [Prestel,SH] arXiv:1506.05057

$$y = \frac{t\,x_a/z_a}{q^2 - s_{ai}},\qquad \tilde{z} = \frac{z_a/x_a}{1-y}\,\frac{q^2}{q^2 - s_{ai}}\;.$$

- Second step now a decay $(ai, k) \to (a, i, k)$ can use CDST algorithm with

$$y' = \left[1 + \frac{z_a}{x_a}\,\frac{q^2}{s_{ai}}\right]^{-1}\;,\qquad \tilde{z}' = x_a$$



$2 \to 4$ phase space, massless FF case

- Phase space factorization derived similar to [Dittmaier] hep-ph/9904440
$\to$ s-channel factorization over p_{aij}, subsequently over p_{ai}

$$\int \mathrm{d}\Phi(p_a, p_i, p_j, p_k | q) = \int \frac{\mathrm{d}s_{aij}}{2\pi} \int \mathrm{d}\Phi(p_{aij}, p_k | q) \int \mathrm{d}\Phi(p_a, p_i, p_j | p_{aij})$$
$$= \int \mathrm{d}\Phi(\tilde{p}_{aij}, \tilde{p}_k | q) \int \Big[\mathrm{d}\Phi(p_a, p_i, p_j | \tilde{p}_{aij}, \tilde{p}_k)\Big]$$

- Nearly reduces to iterated $2 \to 3$ phase space

$$\int \Big[\mathrm{d}\Phi(p_a, p_i, p_j | \tilde{p}_{aij}, \tilde{p}_k)\Big] =$$
$$\underbrace{\frac{1}{4(2\pi)^3} \int \frac{\mathrm{d}t}{t} \int \mathrm{d}z_a \int \mathrm{d}\phi_j}_{\text{emission of } j} \underbrace{\frac{1}{4(2\pi)^3} \int \mathrm{d}s_{ai} \int \frac{\mathrm{d}x_a}{x_a} \int \mathrm{d}\phi_i}_{\text{emission of } i} \, 2\, p_{ai} p_j$$

- Fully massive case worked out for all dipoles [Prestel,SH] arXiv:1705.00742

Collinear factorization and resummation

- Combination with massless matrix element in collinear limit leads to

$$\int \mathrm{d}\Phi(p_a,p_i,p_j,p_k|\,q)\,|M_{n+2}(a,i,j,k|\,q)|^2$$
$$= \int \frac{\mathrm{d}t}{t}\int \mathrm{d}z_a \int \mathrm{d}s_{ai}\int \frac{\mathrm{d}x_a}{x_a}\int \frac{\mathrm{d}\phi_i}{2\pi}\,\frac{2p_{ai}p_j}{s_{aij}}$$
$$\times\left(\frac{\alpha_s}{2\pi}\right)^2\,\frac{P_{(aij)a}(p_a,p_i,p_j)}{s_{aij}}\int \mathrm{d}\Phi(\tilde{p}_{aij},\tilde{p}_k|\,q)\,|M_n(\widetilde{aij},\tilde{k}|\,q)|^2$$

- Write as differential branching probability

$$\frac{\mathrm{d}\ln\Delta^{1\to3}_{(aij)a}}{\mathrm{d}\ln t} = \int \mathrm{d}z_a\,\frac{z_a z_i}{1-z_a}\int \mathrm{d}s_{ai}\int \frac{\mathrm{d}x_a}{x_a}\int \frac{\mathrm{d}\phi_i}{2\pi}\,\left(\frac{\alpha_s}{2\pi}\right)^2\,\frac{P_{(aij)a}(p_a,p_i,p_j)}{s_{aij}^2/\,2\,p_{ai}p_j}$$

- LO PS accounts for iterated collinear limit, hence we must subtract

$$\frac{\mathrm{d}\ln\Delta^{(1\to2)^2}_{(aij)a}}{\mathrm{d}\ln t} = \int \mathrm{d}z_a\,\frac{z_a z_i}{1-z_a}\int \frac{\mathrm{d}s_{ai}}{s_{ai}}\int \frac{\mathrm{d}\xi}{\xi}\,\left(\frac{\alpha_s}{2\pi}\right)^2\,\frac{\sum_{(ai)}P^{(0)}_{(aij)(ai)}(\xi)P^{(0)}_{(ai)a}(z_a/\xi)}{s_{aij}/\,2\,p_{ai}p_j}$$

Collinear factorization and resummation

- Simplest possible configuration $q \to q'$ [Catani,Grazzini] hep-ph/9908523

$$P_{qq'} = \frac{1}{2} C_F T_R \frac{s_{aij}}{s_{ai}} \left[-\frac{t_{ai,j}^2}{s_{ai} s_{aij}} + \frac{4\, z_j + (z_a - z_i)^2}{z_a + z_i} + (1 - 2\varepsilon) \left(z_a + z_i - \frac{s_{ai}}{s_{aij}} \right) \right]$$

where $(z_a + z_i)\, t_{ai,j} = 2(z_a s_{ij} - z_i s_{aj}) + (z_a - z_i) s_{ai}$

- Apparent collinear singularity in s_{ai} that cancels upon azimuthal averaging against iterated LO splitting
- But integrand locally divergent $\to$ Not amenable to MC simulation
- Solved by subtraction of spin-correlated LO splitting functions [Somogyi,Trocsanyi,del Duca] hep-ph/0502226

$$P_{qg}^{\mu\nu} = C_F \left[-2 \frac{z}{1-z} \frac{k_T^\mu k_T^\nu}{k_T^2} + \frac{1-z}{2} \left(-g^{\mu\nu} + \frac{p^\mu n^\nu + p^\nu n^\mu}{np} \right) \right]$$

$$P_{gq}^{\mu\nu} = T_R \left[-g^{\mu\nu} + 4\, z(1-z) \frac{k_T^\mu k_T^\nu}{k_T^2} \right]$$

- Leads to additional subtraction term

$$\Delta P_{qq'} = C_F T_R \frac{4 z_a z_i z_j}{(1 - z_j)^3} \left(1 - 2 \cos^2 \phi \right) , \qquad \cos\phi = \frac{s_{ai} s_{jk} + s_{ak} s_{ij} - s_{aj} s_{ik}}{\sqrt{4\, s_{ai} s_{ak}\, s_{ij} s_{jk}}}$$

Collinear factorization and resummation

- Reference for $q \to q'$ upon integration over s_{ai}, x_a, ϕ_j given by NLO kernel

$$P_{qq'}(z) = C_F T_R \left[(1+z)\ln^2 z - \left(\frac{8}{3}z^2 + 9z + 5 \right) \ln z + \frac{56}{9}z^2 + 4z - 8 - \frac{20}{9z} \right]$$

- So far we only have

$$P_{qq'}(z) = -C_F T_R \Big[5(1-z) + 2(1+z)\ln z \Big]$$

- Remainder scheme-dependent, must be computed in D dimensions
- Key is to realize that we just set up a local, modified subtraction method

$$\boldsymbol{P_{qq'}(z) = \left(\mathrm{I} + \frac{1}{\varepsilon}\mathcal{P} - \mathcal{I} \right)_{qq'}(z) + \int \mathrm{d}\Phi_{+1} (\mathrm{R} - \mathrm{S})_{qq'}(z, \Phi_{+1})}$$

where

$$\mathrm{I}_{qq'}(z) = \int \mathrm{d}\Phi_{+1} S_{qq'}(z, \Phi_{+1}) \,, \qquad \mathcal{P}_{qq'}(z) = \int_z \frac{\mathrm{d}x}{x} P^{(0)}_{qg}(x) P^{(0)}_{gq}(z/x)$$

$$\mathcal{I}_{qq'}(z) = 2\int_z \frac{\mathrm{d}x}{x} C_F \left(\frac{1+(1-x)^2}{x} \ln(x(1-x)) + x \right) P_{gq}(z/x)$$

Collinear factorization and resummation

- Analytical computation of I not needed, as $\text{I} + \mathcal{P}/\varepsilon$ finite
- Simulate as endpoint, starting from $\mathcal{O}(\varepsilon)$ coefficient of integrand
  - Generate point in triple collinear phase space, but retroactively project onto $s_{ai} = 0$
  - Guarantees phase-space coverage identical to fully differential simulation
- Kernel for endpoint contribution defined by $\Delta\text{I}_{qq'} = \tilde{\text{I}}_{qq'} - \tilde{\mathcal{I}}_{qq'}$, where

$$\tilde{\text{I}}_{qq'} = C_F T_R \left[\frac{1+z_j^2}{1-z_j} + \left(1 - \frac{2\,z_a z_i}{(z_a+z_i)^2}\right)\left(1 - z_j + \frac{1+z_j^2}{1-z_j}\right)\left(\ln(z_a z_i z_j) - 1\right)\right]$$

$$\tilde{\mathcal{I}}_{qq'} = 2C_F \left[\frac{1+z_j^2}{1-z_j} \ln((z_a+z_i)\,z_j) + (1-z_j)\right] P_{gq}^{(0)}\left(\frac{z_a}{z_a+z_i}\right) .$$

- Cross-checked method analytically using phase space from
  [Gehrmann,Gehrmann-DeRidder,Heinrich] hep-ph/0311276 (timelike)
  [Ellis,Vogelsang] hep-ph/9602356 (spacelike)

Numerical implementation in Dire

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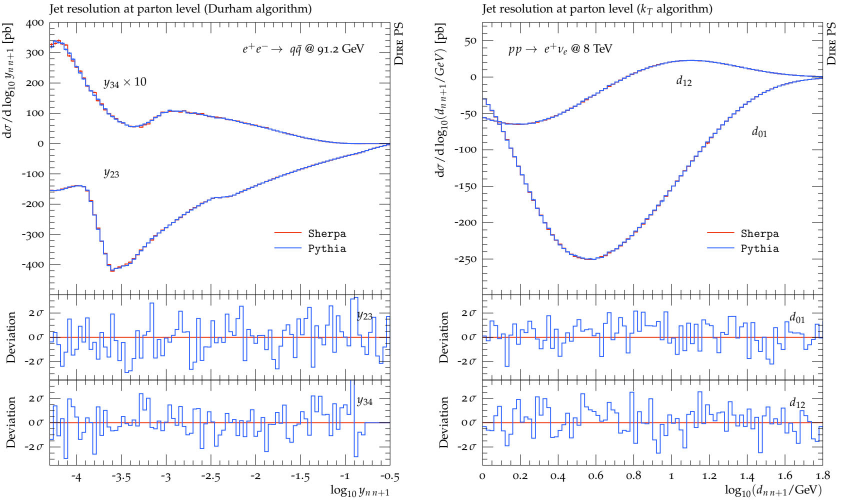
Basic layout of Dire [Prestel,SH] arXiv:1506.05057

- Dipole-like parton shower, kernels as close as possible to DGLAP
- Partial fraction soft eikonal à la Catani-Seymour, evolve in dipole-k_T
- Two independent implementations (Pythia & Sherpa)
- Cross-validation at particle level

New developments

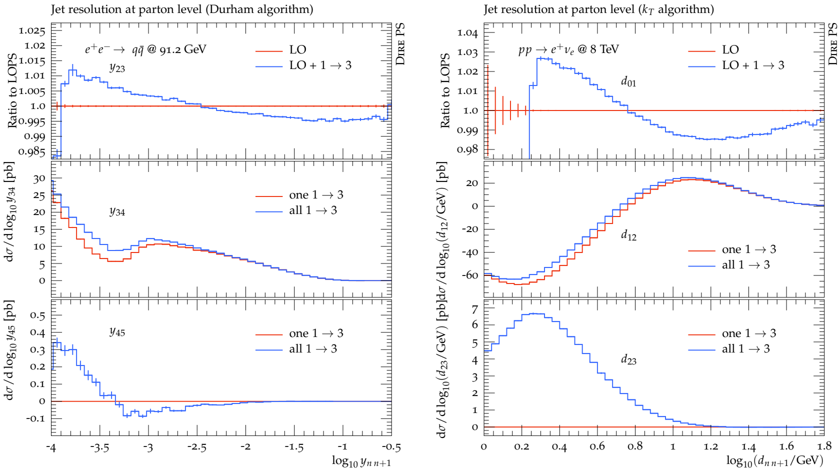
- MC counterterms implemented in Amegic & Comix [SH]
- MC@NLO matching & NLO subtraction in Sherpa [SH]
- UNLOPS / MEPS@NLO merging in Pythia / Sherpa [Prestel,SH]
- Flavor-changing triple collinear splitting functions [Prestel,SH] arXiv:1507.00742
- NLO DGLAP kernels & 3-loop cusp [Krauss,Prestel,SH] arXiv:1507.00982

Validation



- Effect of single $1 \to 3$ emission on leading and next-to-leading jet rate

Impact on leading-order PS prediction

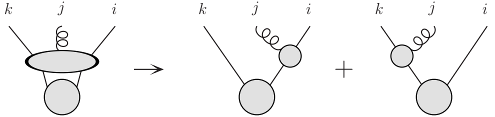


- Effect of $1 \to 3$ emissions on leading jet rate
- Impact of multiple $1 \to 3$ emissions

Soft-gluon radiation at LO

- Soft eikonal partial fractioned to account for coherence effects (3-parton correlations) [Catani,Seymour] hep-ph/9605323

$$\frac{p_ip_k}{(p_ip_j)(p_jp_k)} \to \frac{1}{p_ip_j}\frac{p_ip_k}{(p_i+p_k)p_j} + \frac{1}{p_kp_j}\frac{p_ip_k}{(p_i+p_k)p_j}$$



- "Spectator"-dependent kernels, singular in soft-collinear region only
  → Soft enhanced term of splitting functions replaced as

$$2C_a\,\frac{1}{1-z} \to 2C_a\,\frac{1-z}{(1-z)^2+\kappa^2_{j,ik}} \qquad \kappa^2_{j,ik} = \frac{k^2_{\perp,j\leftrightarrow ik}}{Q^2} = \frac{4\,p_ip_j\,p_jp_k}{Q^4}$$

- For correct soft FS evolution, color correlations must be respected

$$P_{gg} \to P_{gg}(1-z_j,\kappa^2_{j,ik}) + P_{gg}(1-z_i,\kappa^2_{i,jk})$$

$$P_{gg}(z,\,\kappa^2) = 2C_A\left[\frac{1-z}{(1-z)^2+\kappa^2} - 1 + \frac{z(1-z)}{2}\right]$$

Flavor non-changing splittings at NLO

- Two-loop cusp anomalous dimension included at LO in CMW prescription [Catani,Marchesini,Webber] NPB349(1991)635

$$P_{ab}^{(0)} \to P_{ab}^{(0)} + \delta_{ab}\, 2C_a \frac{1-z}{(1-z)^2+\kappa^2}\, \frac{\alpha_s}{2\pi}\Gamma^{(2)}\,, \quad \Gamma^{(2)} = \left[\frac{67}{18} - \frac{\pi^2}{6}\right] C_A - \frac{10}{9} T_F$$

  Can add structurally identical 3-loop term [Moch,Vermaseren,Vogt] hep-ph/0403192

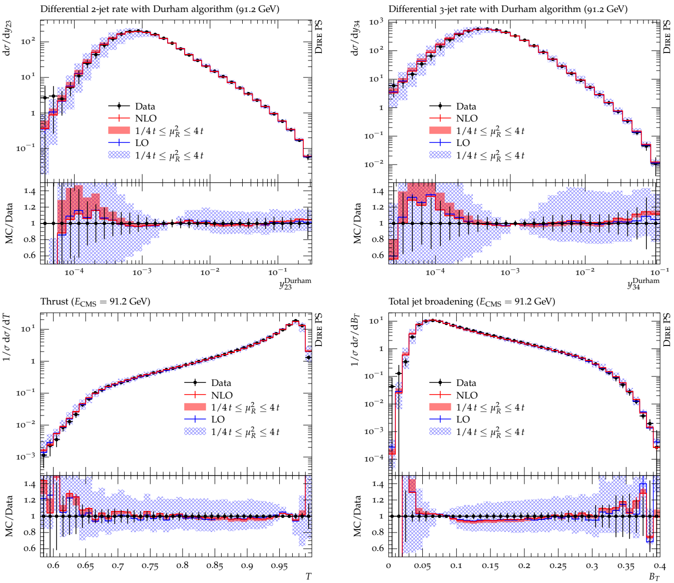
- Subtract $\Gamma^{(2)}$ from 2-loop splitting function to avoid double-counting

$$P_{ab}^{(1)}(z,\kappa^2) \to P_{ab}^{(1)}(z) - \delta_{ab}\, \frac{2C_a}{1-z}\, \Gamma^{(2)}$$

- Scale variation on soft-enhanced part of splitting functions relies on [Amati,Bassetto,Ciafaloni,Marchesini,Veneziano] NB173(1980)429 (renormalization scale set by $k_T^2 \leftrightarrow$ limit on gluon virtuality)

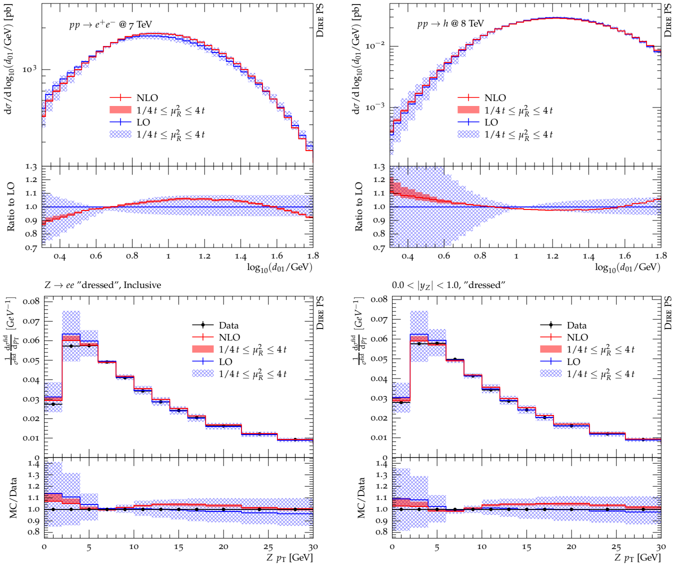
First results

[Krauss,Prestel,SH] arXiv:1705.00982



First results

[Krauss,Prestel,SH] arXiv:1705.00982



Outlook

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Summary

- Developed MC algorithm to implement $2 \to 4$ splittings that recovers integrated NLO splitting functions for $q \to q'$ / $q \to \bar{q}$
- Cross-validated implementation Pythia $\leftrightarrow$ Sherpa
- Flavor non-changing splitting kernels included in integrated form

Next steps

- Extension of differential simulation to flavor non-changing kernels
- Simulation of soft-gluon radiation at NLO (single emission)
- Color correlations in soft-gluon emissions (multiple emissions)

Possible benefits

- Parton showers with more realistic uncertainty estimates
- Comparison to analytic resummation at higher precision
- Same shower algorithm in two different generators

Thank you for your attention

Backup slides

Negative branching "probabilities" in Markov Chains

- Problem in NLO splitting kernels, sub-leading color terms, etc. lies in negative weights $\rightarrow$ no-emission probability *locally* exceeds unity
- Recall standard veto algorithm: $\mathcal{P}_{\rm no}(t,t') = \exp\{F(t) - F(t')\}$
  Exact MC solution $t = F^{-1}\big[\,F(t') + \ln R\,\big]$, R – random number
- Don't want or can't compute $F(t) = -\int_t {\rm d}\bar{t} f(\bar{t})$,
  instead find simple function $g(t) > f(t)$ with integral $G(t)$
- Generate points according to $g(t)$ and accept with $f(t)/g(t)$

Standard probability for one acceptance with n rejections

$$\frac{f(t)}{g(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} {\rm d}\bar{t}\, g(\bar{t})\right\} \prod_{i=1}^{n} \left[\int_{t_{i-1}}^{t'} {\rm d}t_i \left(1 - \frac{f(t_i)}{g(t_i)}\right) g(t_i)\, \exp\left\{-\int_{t_i}^{t_{i+1}} {\rm d}\bar{t}\, g(\bar{t})\right\}\right]$$

Split weight into MC and analytic part using auxiliary function $h(t)$

$$\frac{f(t)}{h(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} {\rm d}\bar{t}\, g(\bar{t})\right\} \prod_{i=1}^{n} \left[\int_{t_{i-1}}^{t'} {\rm d}t_i \left(1 - \frac{f(t_i)}{h(t_i)}\right) g(t_i)\, \exp\left\{-\int_{t_i}^{t_{i+1}} {\rm d}\bar{t}\, g(\bar{t})\right\}\right]$$

$$w(t, t_1, \ldots, t_n) = \frac{h(t)}{g(t)} \prod_{i=1}^{n} \frac{h(t_i)}{g(t_i)} \frac{g(t_i) - f(t_i)}{h(t_i) - f(t_i)}$$

Negative branching "probabilities" in Markov Chains

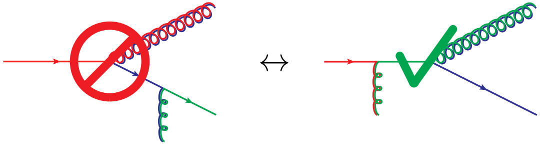
Looks trivial, surprisingly it's not: It allows to

- Resum sub-leading color terms in MC@NLO and POWHEG
  [Krauss,Schönherr,Siegert,SH] arXiv:1111.1220
- Implement triple-collinear splitting functions in parton showers
  [Prestel,SH] arXiv:1705.00742
- Use PDFs with negative values in parton showers
  [Prestel,SH] arXiv:1506.05057
- Enhance branching probabilities in parton showers
  [Schumann,Siegert,SH] arXiv:0912.3501, [Lönnblad] arXiv:1211.7204
- Reweight parton showers [Bellm,Plätzer,Richardson,Siódmok,Webster] arXiv:1605.08256
  [Mrenna,Skands] arXiv:1605.08352, [Bothmann,Schönherr,Schumann] arXiv:1606.08753

Color coherence and the dipole picture

[Marchesini,Webber] NPB310(1988)461

- Individual color charges inside a color dipole cannot be resolved by gluons of wavelength larger than the dipole size
  $\rightarrow$ emission off combined mother parton instead

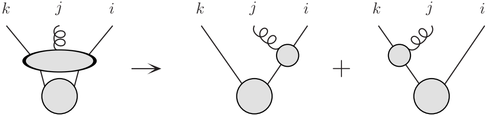


- Net effect is destructive interference outside cone with opening angle defined by emitting color dipole
  $\rightarrow$ Soft anomalous dimension halved due to reduced phase space
- Formerly implemented by angular ordering / angular veto
  [Webber at al.] hep-ph/0210213, [Sjöstrand et al.] hep-ph/0603175
- Alternative description in terms of color dipoles
  [Gustafsson,Pettersson] NPB306(1988)746, [Kharraziha,Lönnblad] hep-ph/9709424
  [Winter,Krauss] arXiv:0712.3913

The midpoint between dipole and parton showers

- Angular ordered / vetoed parton shower does not fill full phase space
  Dipole shower lacks parton interpretation $\to$ prefer alternative to both
- Can preserve parton picture by partial fractioning soft eikonal
  $\leftrightarrow$ soft enhanced part of splitting function [Catani,Seymour] hep-ph/9605323

$$\frac{p_i p_k}{(p_i p_j)(p_j p_k)} \to \frac{1}{p_i p_j}\,\frac{p_i p_k}{(p_i+p_k)p_j} + \frac{1}{p_k p_j}\,\frac{p_i p_k}{(p_i+p_k)p_j}$$



- "Spectator"-dependent kernels, singular in soft-collinear region only
  $\to$ capture dominant coherence effects (3-parton correlations)

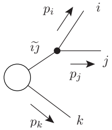
$$\frac{1}{1-z} \to \frac{1-z}{(1-z)^2+\kappa^2} \qquad \kappa^2 = \frac{k_\perp^2}{Q^2}$$

- For correct soft evolution, ordering variable must be identical
  at both "dipole ends" ($\to$ recover soft eikonal at integrand level)

The midpoint between dipole and parton showers

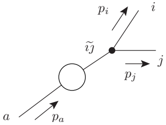
Choose parametrization such that soft term is $\frac{1-z}{(1-z)^2+\kappa^2}$ in all dipole types

(1) FF



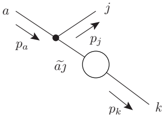
$$\kappa^2 = \frac{p_i p_j \, p_j p_k}{(p_{\widetilde{ij}} p_{\tilde{k}})^2} \qquad z_j = \frac{p_j p_k}{p_{\widetilde{ij}} p_{\tilde{k}}}$$

(2) FI



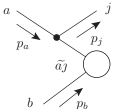
$$\kappa^2 = \frac{p_i p_j \, p_j p_a}{(p_{ij} p_a)^2} \qquad z_j = \frac{p_j p_a}{p_{ij} p_a}$$

(3) IF



$$\kappa^2 = \frac{p_a p_j \, p_j p_k}{(p_{jk} p_a)^2} \qquad z_j = \frac{p_j p_k}{p_{jk} p_a}$$

(4) II



$$\kappa^2 = \frac{p_a p_j \, p_j p_b}{(p_a p_b)^2} \qquad z_j = \frac{p_j p_b}{p_a p_b}$$

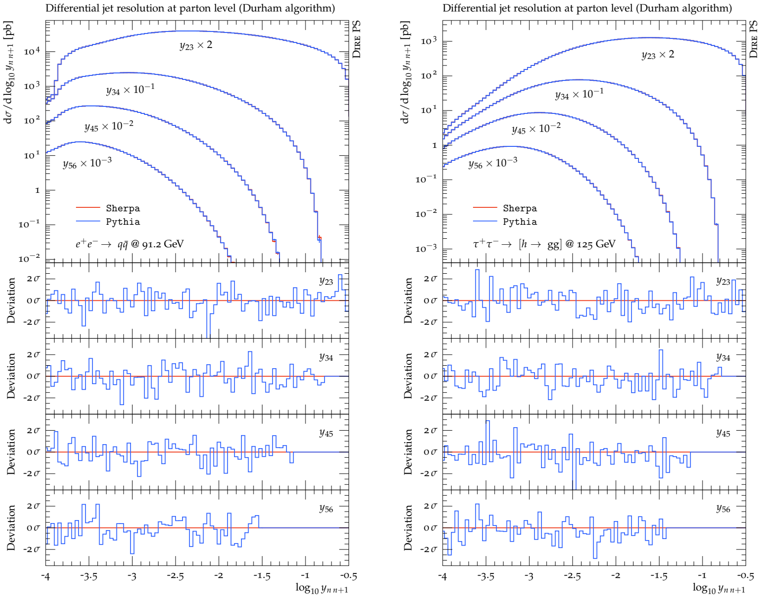
Preserve collinear anomalous dimensions & sum rules $\rightarrow$ splitting functions fixed

$$P_{qq}(z,\kappa^2) = 2\,C_F \left[\left(\frac{1-z}{(1-z)^2+\kappa^2} \right)_+ - \frac{1+z}{2} \right] + \gamma_q \, \delta(1-z)$$

$$P_{gg}(z,\kappa^2) = 2\,C_A \left[\left(\frac{1-z}{(1-z)^2+\kappa^2} \right)_+ + \frac{z}{z^2+\kappa^2} - 2 + z(1-z) \right] + \gamma_g \, \delta(1-z)$$

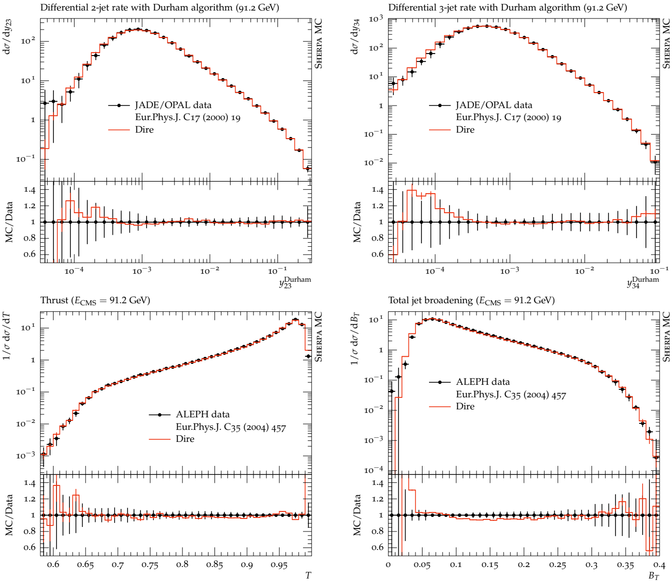
$$P_{qg}(z,\kappa^2) = 2\,C_F \left[\frac{z}{z^2+\kappa^2} - \frac{2-z}{2} \right] \qquad P_{gq}(z,\kappa^2) = T_R \left[z^2 + (1-z)^2 \right]$$

Validation in $e^+e^- \to$ hadrons



Comparison to LEP data

[Prestel,SH] arXiv:1506.05057



Dire as an NLO subtraction method and MC@NLO

[SH] TBP?

- Can view new shower model as modification of CS subtraction
- IR counterterms computed and implemented in Sherpa (improved cancellation in $pp \to h + j$ due to regulated $1/z$ terms)
- Sherpa MC@NLO based on exponentiation of CS dipole subtraction terms
  [Krauss,Siegert,Schönherr,SH]
  arXiv:1111.1220, arXiv:1208.2815
- Dire modified CS subtraction automatically available for MC@NLO matching
- Interesting differences due to evolution variables and kernels