The Elusive Parton Shower Uncertainty

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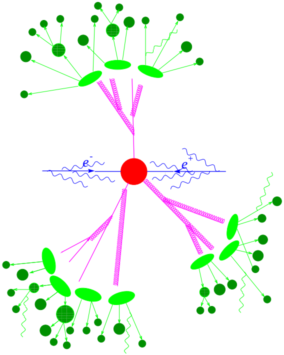
Parton Showers for future e+e- colliders

CERN, 26/04/2023

Event generators for e⁺e⁻ colliders

[Buckley et al.] arXiv:1101.2599
[Campbell et al.] arXiv:2203.11110

- Short distance interactions
  - Signal process
  - Radiative corrections
- Long-distance interactions
  - Hadronization
  - Particle decays



Divide and Conquer

- Quantity of interest: Total interaction rate
- Convolution of short & long distance physics

$$\sigma_{ee\to h+X} = \sum_{i\in\{q,g\}} \int \mathrm{d}x\, \underbrace{\hat{\sigma}_{ee\to i+X}(x,\mu_F^2)}_{\text{short distance}}\, \underbrace{D_i^{(h)}(x,\mu_F^2)}_{\text{long distance}}$$

Connection to QCD theory

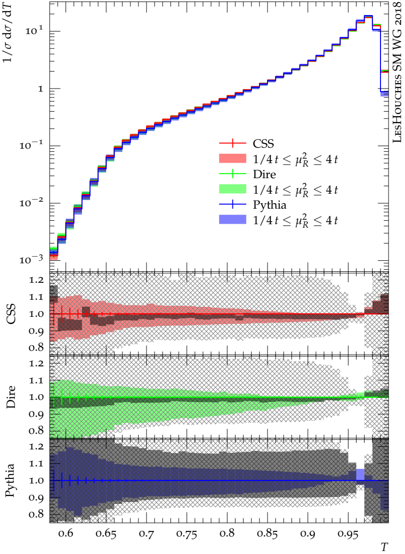
- $\hat{\sigma}_{ee\to i+X}(\mu_F^2) \to$ Collinearly factorized fixed-order result at N^{x}LO
  Implemented in fully differential form to be maximally useful
- $D_i^{(h)}(x,\mu_F^2) \to$ Collinearly factorized fragmentation function at N^{y}LO
  Evaluated at $O(1\text{GeV}^2)$ and expanded into a series above 1GeV^2
  DGLAP: $\frac{\mathrm{d}x\,xD_a(x,t)}{\mathrm{d}\ln t} = \sum_{b=q,g}\int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}z\,\frac{\alpha_s}{2\pi}\big[zP_{ab}(z)\big]_+ \tau D_b(\tau,t)\,\delta(x-\tau z)$
  Implemented by parton showers, dipole showers, antenna showers, ...
  - In the spotlight recently due to missing systematic error budget
    - Logarithmic precision [PanScales,Deductor,Herwig,Sherpa,...]
    - Higher-order corrections [Vincia,Sherpa,Herwig,...]
  - Uncontrollable effects from on-shell kinematics mapping
  - Various possibilities of matching to collinear limit

$$P_{aa}(z) = C_a\frac{2z}{1-z} + \ldots \qquad \leftrightarrow \qquad J^\mu = \sum_i \mathbf{T}_i\frac{p_i^\mu}{p_ip_j}$$

How to quantify parton-shower uncertainties

[LesHouches] arXiv:1605.04692, arXiv:1803.07977

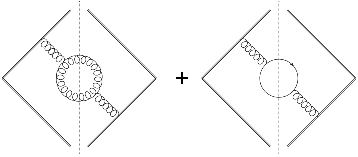
- First systematic attempt to estimate PS variations by MCnet groups at LesHouches 2015/2017 $\rightarrow$
- Renormalization scale uncertainties based on LO splitting kernels and their order α_s^2 soft corrections
- Only variations in probability density
- Kinematics and evolution variable remain similar among contenders



Is this in any way representative or systematic?

Approximate scale uncertainties – soft-collinear NLO

- Leading higher-order corrections to soft-gluon effects from collinear decay



$$+ \ \dots \ = \sum_{b=q,g} j_{ij,\mu}(p_{12}) j_{ij,\nu}(p_{12}) \, \frac{P^{\mu\nu}_{gb}(z_1)}{s_{12}}$$

$$P^{\mu\nu}_{gq}(z) = T_R \left(-g^{\mu\nu} + 4\, z(1-z)\, \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \right)$$

$$P^{\mu\nu}_{gg}(z) = C_A \left(-g^{\mu\nu} \left(\frac{z}{1-z} + \frac{1-z}{z} \right) - 2\,(1-\varepsilon) z(1-z)\, \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \right)$$

- Combine with phase space for one parton emission in collinear limit
  $D = 4 - 2\varepsilon$, $y = s_{12}/Q^2$, see for example [Catani,Seymour] hep-ph/9605323

$$\mathrm{d}\Phi_{+1} = \frac{Q^{2-2\varepsilon}}{16\pi^2} \frac{(4\pi)^\varepsilon}{\Gamma(1-\varepsilon)} \, \mathrm{d}y \, \mathrm{d}z \, \big[y\, z(1-z) \big]^{-\varepsilon}$$

- Perform Laurent series expansion

$$\frac{1}{y^{1+\varepsilon}} = -\frac{\delta(y)}{\varepsilon} + \sum_{n=0}^{\infty} \frac{\varepsilon^n}{n!} \left(\frac{\ln^n y}{y} \right)_+$$

Approximate scale uncertainties – soft-collinear NLO

- $\mathcal{O}(\varepsilon^0)$ remainder terms proportional to

$$g \to q\bar{q}: \quad T_R \Big[2z(1-z) + \big(1 - 2z(1-z)\big)\ln(z(1-z))\Big]$$

$$g \to gg: \quad 2C_A \left[\frac{\ln z}{1-z} + \frac{\ln(1-z)}{z} + \big(-2 + z(1-z)\big)\ln(z(1-z))\right]$$

- Integration over z & some additional semi-classical terms $\rightarrow$

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{10}{9} T_R n_f$$

  - Local K-factor for soft-gluon emission
  - Scheme dependent: originates in dim. reg. $\times$ $\overline{\mathrm{MS}}$
  - Origin of CMW scheme [Catani,Marchesini,Webber] NPB349(1991)635

- Similar logic leads to logarithmic contribution

$$-\beta_0 \log \frac{m_T^2}{\mu^2}$$

- Soft splitting function with estimated 2-loop corrections

$$P_{aa}(z) \xrightarrow{z\to 1} \frac{2C_a}{1-z}\left[1 + \frac{\alpha_s(\mu^2)}{2\pi}\left(-\beta_0 \ln \frac{k_T^2}{\mu^2} + K\right)\right]$$

Recoil effects at finite resolution scale

- Logarithmic accuracy of parton shower can be quantified by comparing results to (semi-)analytic resummation e.g. [Banfi,Salam,Zanderighi] hep-ph/0407286
- Example: Thrust or FC_0 in $e^+e^- \to$hadrons

- Define a shower evolution variable $\xi = k_T^2/(1-z)$
- Parton-shower one-emission probability for $\xi > Q^2\tau$

$$R_{\rm PS}(\tau) = 2\int_{Q^2\tau}^{Q^2}\frac{d\xi}{\xi}\int_{z_{\rm min}}^{z_{\rm max}} dz\,\frac{\alpha_s\big(k_T^2\big)}{2\pi}\,C_F\left[\frac{2}{1-z}-(1+z)\right]\Theta(\eta)$$

- Approximate to NLL accuracy

$$R_{\rm NLL}(\tau) = 2\int_{Q^2\tau}^{Q^2}\frac{d\xi}{\xi}\left[\int_0^1 dz\,\frac{\alpha_s\big(k_T^2\big)}{2\pi}\,\frac{2\,C_F}{1-z}\Theta(\eta)-\frac{\alpha_s(\xi)}{\pi}\,C_F B_q\right]$$

Recoil effects at finite resolution scale

- Cumulative cross section $\Sigma(\tau) = e^{-R(\tau)}\mathcal{F}(\tau)$ obtained from all-orders resummed result by Taylor expansion of virtual corrections in cutoff ε

$$\mathcal{F}(\tau) = \int \mathrm{d}^3k_1|M(k_1)|^2\, e^{-R'\ln\frac{\tau}{\varepsilon v_1}} \sum_{m=0}^{\infty}\frac{1}{m!}\left(\prod_{i=2}^{m+1}\int_{\varepsilon v_1}^{v_1}\mathrm{d}^3k_i|M(k_i)|^2\right) \times \Theta\big(\tau - V(\{p\},k_1,\ldots,k_n)\big)$$

  - $\mathcal{F}(\tau)$ is pure NLL & accounts for (correlated) multiple-emission effects
- In order to make $\mathcal{F}(\tau)$ calculable, make the following assumptions
  - Observable is recursively infrared and collinear safe
  - Hold $\alpha_s(Q^2)\ln\tau$ fixed, while taking limit $\tau \to 0$

    $\to$ Can factorize integrals and neglect kinematic edge effects

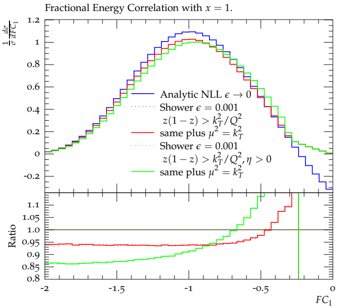
    Can be interpreted as $\alpha_s \to 0$ or $s \to \infty$ limit
- Breaks momentum conservation and unitarity for finite τ

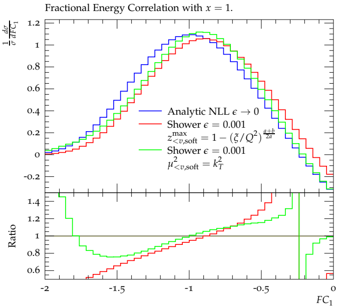
  $\to$ Clean NLL result, but uncontrolled kinematic corrections

  [Reichelt,Siegert,SH] arXiv:1711.03497

Recoil effects at finite resolution scale

[Reichelt,Siegert,SH] arXiv:1711.03497





Single emission effects

- 4-mom conservation
- PS sectorization

Multiple emission effects

- z bounds by unitarity

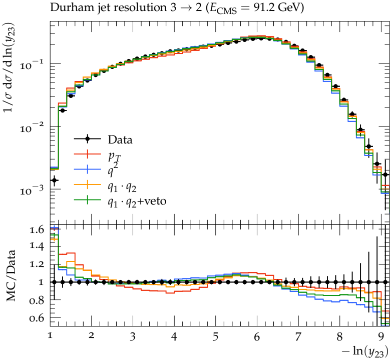
- Sizable differences away from $\tau \to 0$ limit
- Parametric NLL precision leaves ample room for variations of kinematical origin

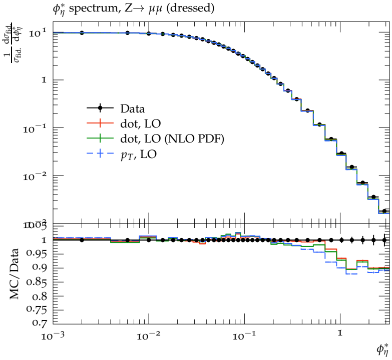
Residual uncertainties

Herwig angular ordered parton showers

[Bewick,Ferrario-Ravasio,Richardson,Seymour] arXiv:1904.11866, arXiv:2107.04051

- Comparison of q_T, q^2 & dot product preserving recoil schemes

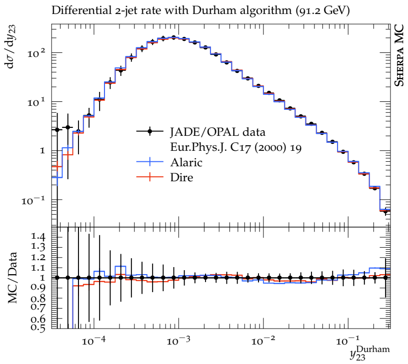


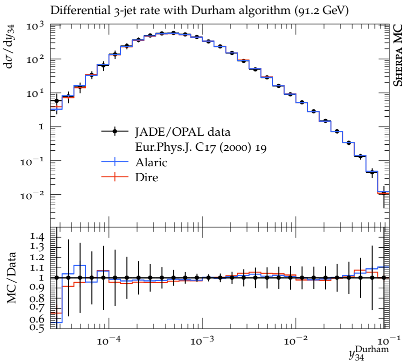


Alaric parton shower

[Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057

- Comparison to experimental data from LEP

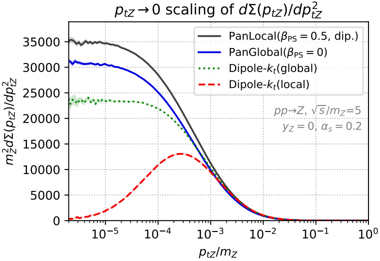


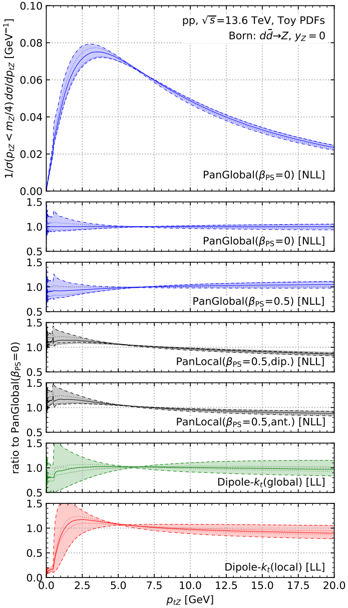


PanScales parton shower

[van Beekveld,Ferrario Ravasio,Hamilton,Salam, Soto-Ontoso,Soyez,Verheyen] arXiv:2207.09467

- Comparison of different PanScales showers all provably NLL accurate
- Toy PDF, fixed flavor initial state
- Conventional dipole schemes do not reproduce [Parisi,Petronzio] NPB154(1979)427



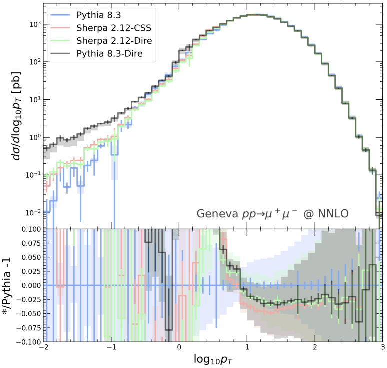


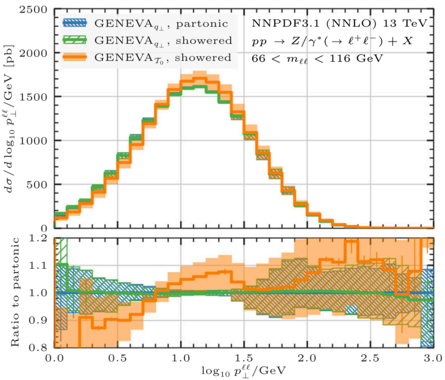
Impact of parton-shower variations on matching

Impact of parton-shower uncertainty on matching

[D. Napoletano] arXiv:2212.10489, [Alioli et al.] arXiv:2102.08390

- NNLO+PS precise predictions for $pp \to Z$ from Geneva
- Matched to shower by vetoing events with $r_N(\Phi_{N+M}) > r_N$



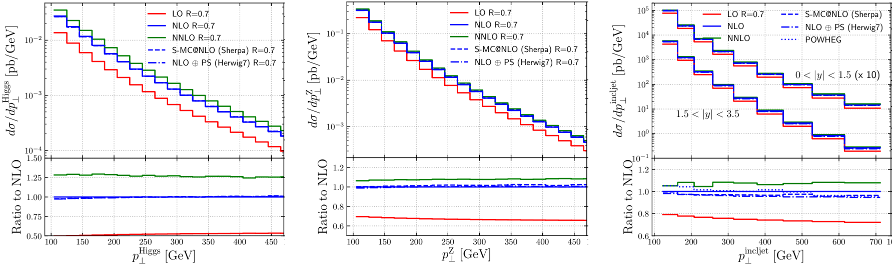


- Parton shower scheme uncertainty
- Choice of resolution variable

Impact of parton-shower uncertainty on matching

[Bellm at al.] arXiv:1903.12563

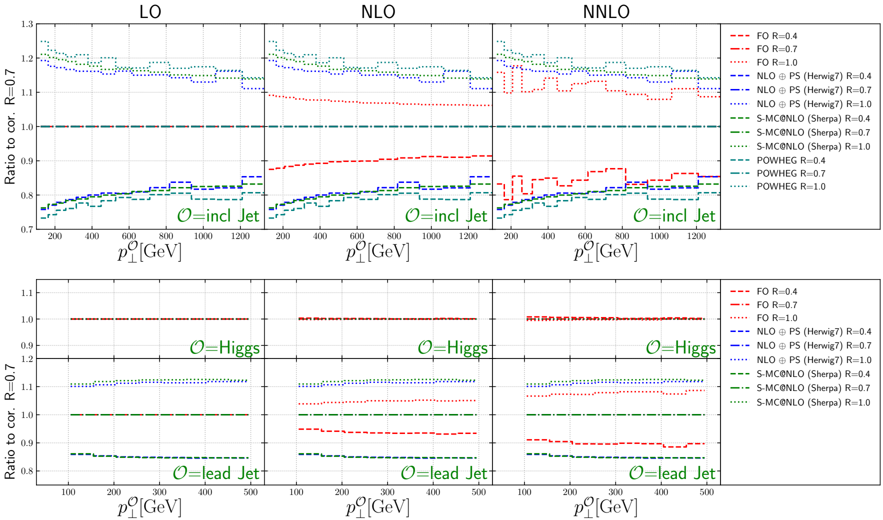
- Ratio of inclusive jet-$p_\perp$ cross sections for different radii in $pp \to jets$



Impact of parton-shower uncertainty on matching

[Bellm at al.] arXiv:1903.12563

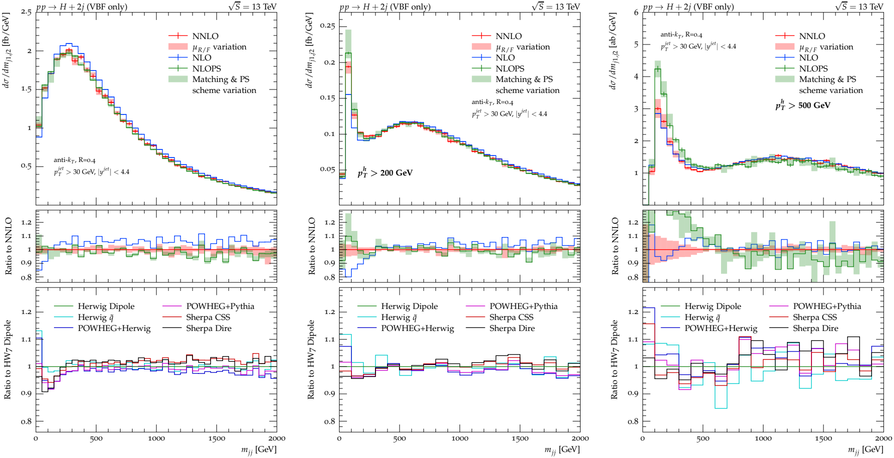
- Ratio of inclusive jet-$p_\perp$ spectra for different radii in $pp \to jj$ / $pp \to H + j$



Impact of parton-shower uncertainty on matching

[Buckley et al.] arXiv:2105.11399

- m_{jj} of two leading jets in VBF Higgs production



Higher-order corrections

Effects beyond the CMW scheme

Complete soft NLO corrections at leading color

[Marchesini,Korchemsky] PLB313(1993)433, hep-ph/9210281

- Need a benchmark for parton shower to reproduce
  $\rightarrow$ soft-gluon resummed expression of Drell-Yan or DIS cross section

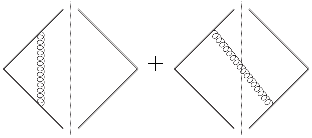
$$\frac{1}{\sigma}\frac{d\sigma(z,Q^2)}{d\log Q^2} = \mathcal{H}(Q^2)\,\widetilde{W}(z,Q^2)$$

  RGE governed by Wilson loop $\widetilde{W}$ ($Q(1-z)$ - total soft gluon energy)
- Non-abelian exponentiation theorem allows to expand as

$$\widetilde{W} = \exp\left\{\sum_{i=1}^{\infty} w^{(\mathrm{n})}\right\}$$

- One-loop result given by

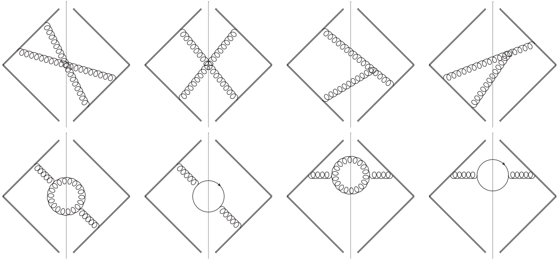
$$w^{(1)} = C_F\,\frac{\alpha_s(\mu)}{2\pi}\left[\ln^2 L + \frac{\pi^2}{6}\right] \qquad \leftrightarrow$$



where $L = -b_+ b_- / b_0^2$ and $b_0 = 2\,e^{-\gamma_E}/\mu$

Complete soft NLO corrections at leading color

- 2-loop contribution $w^{(2)}$ computed from (reals only) [Belitsky] hep-ph/9808389



- Renormalized result in position space

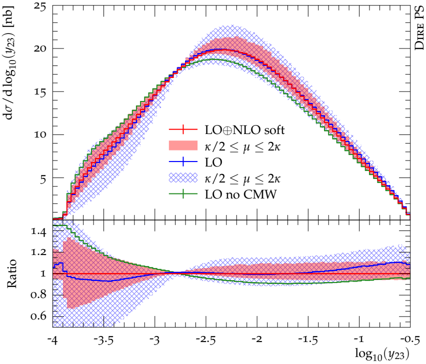
$$w^{(2)} = C_F \frac{\alpha_s^2(\mu)}{(2\pi)^2}\left[-\frac{\beta_0}{6}\ln^3 L + \Gamma^{(2)}_{\rm cusp}\ln^2 L + 2\ln L\left(\Gamma^{(2)}_{\rm soft} + \frac{\pi^2}{12}\beta_0\right) + \ldots\right]$$

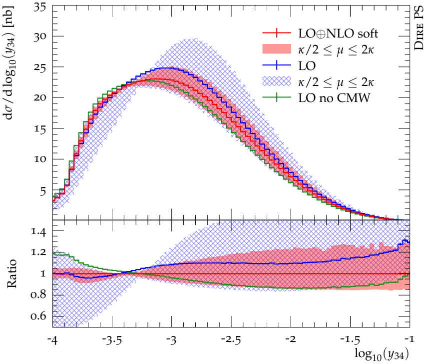
$$\Gamma^{(2)}_{\rm cusp} = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{10}{9} T_R\, n_f\ , \qquad \beta_0 = \frac{11}{6} C_A - \frac{2}{3} T_R n_f$$

$$\Gamma^{(2)}_{\rm soft} = \left(\frac{101}{27} - \frac{11}{72}\pi^2 - \frac{7}{2}\zeta_3\right) C_A - \left(\frac{28}{27} - \frac{\pi^2}{18}\right) T_R\, n_f$$

Complete soft NLO corrections at leading color

[Dulat,Prestel,SH] arXiv:1805.03757





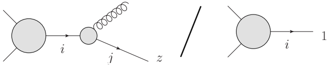
- Impact on $2 \to 3$ and $3 \to 4$ Durham jet rate at LEP I
- Uncertainty bands no longer just estimates
  but perturbative QCD predictions for the first time
- Fair agreement with CMW scheme ($\Gamma_{\text{cusp}}^{(2)}$ contribution)

Collinear NLO corrections

- Higher-order DGLAP evolution kernels obtained from factorization

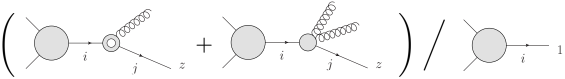
$$D_{ji}^{(0)}(z,\mu) = \delta_{ij}\delta(1-z) \qquad \leftrightarrow$$

$$D_{ji}^{(1)}(z,\mu) = -\frac{1}{\varepsilon}P_{ji}^{(0)}(z) \qquad \leftrightarrow$$



$$D_{ji}^{(2)}(z,\mu) = -\frac{1}{2\varepsilon}P_{ji}^{(1)}(z) + \frac{\beta_0}{4\varepsilon^2}P_{ji}^{(0)}(z) + \frac{1}{2\varepsilon^2}\int_z^1 \frac{\mathrm{d}x}{x}P_{jk}^{(0)}(x)P_{ki}^{(0)}(z/x)$$

$\leftrightarrow$



- $P_{ji}^{(n)}$ not probabilities, but sum rules hold ($\leftrightarrow$ unitarity constraint)
  In particular: Momentum sum rule identical between LO & NLO
- Can perform the NLO computation of $P_{ji}^{(1)}$ fully differentially
  using modified dipole subtraction [Catani,Seymour] hep-ph/9605323

Collinear NLO corrections

[Prestel,SH] arXiv:1705.00742

- Example: Flavor-changing NLO splitting functions

$$P^{(1)}_{qq'}(z) = \mathrm{C}_{qq'}(z) + \mathrm{I}_{qq'}(z) + \int \mathrm{d}\Phi_{+1}\Big[\mathrm{R}_{qq'}(z,\Phi_{+1}) - \mathrm{S}_{qq'}(z,\Phi_{+1})\Big]$$

- Real correction $R_{qq'}$ and subtraction terms $S_{qq'}$
  Difference finite in 4 dimensions $\rightarrow$ amenable to MC simulation
- Integrated subtraction term and factorization counterterm given by

$$\mathrm{I}_{qq'}(z) = \int \mathrm{d}\Phi_{+1} S_{qq'}(z,\Phi_{+1})$$

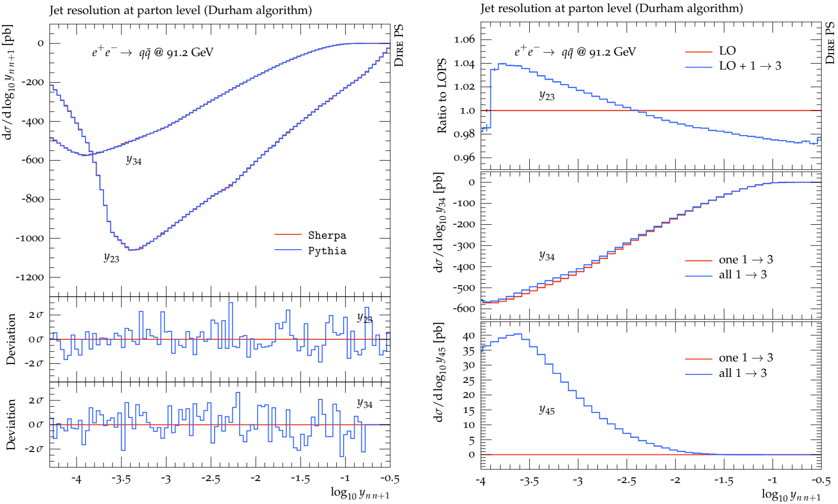
$$\mathrm{C}_{qq'}(z) = \int_z \frac{\mathrm{d}x}{x}\left(P^{(0)}_{qg}(x) + \varepsilon\mathcal{J}^{(1)}_{qg}(x)\right)\frac{1}{\varepsilon}P^{(0)}_{gq}(z/x)$$

$$\mathcal{J}^{(1)}_{qg}(z) = 2C_F\left(\frac{1+(1-x)^2}{x}\ln(x(1-x)) + x\right)$$

- Analytical computation of I not needed, as $\mathrm{I} + \mathcal{P}/\varepsilon$ finite
  generate as endpoint at $s_{ai} = 0$, starting from integrand at $\mathcal{O}(\varepsilon)$
- All components of $P^{(1)}_{qq'}$ eventually finite in 4 dimensions
  Can be simulated fully differentially in parton shower

Collinear NLO corrections

[Gellersen,Prestel,SH] arXiv:2110.05964



- Effects on jet rates in $e^+e^- \to$hadrons at LEP

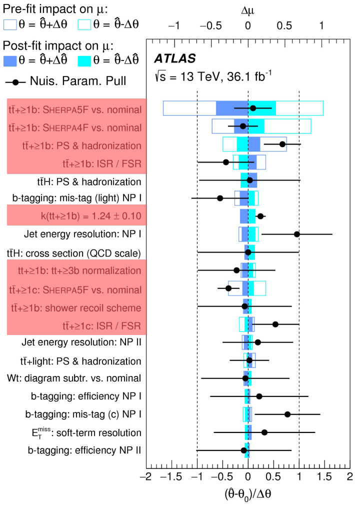
Here be dragons

Heavy flavor production & evolution

$t\bar{t}b\bar{b}$ as an example

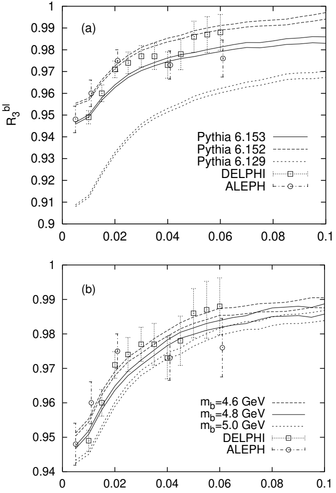
- MC single largest source of uncertainty on extracted signal strength
- Despite intense study of HF production
  - Fixed order, NLL, FONLL
    [Cacciari,Frixione,Houdeau,Mangano,Nason,Ridolfi,...] arXiv:1205.6344, hep-ph/0312132, hep-ph/9801375, NPB373(1992)295
  - In context of particle-level Monte Carlo
    [Norrbin,Sjöstrand], hep-ph/0010012, [Gieseke,Stephens,Webber] hep-ph/0310083, [Schumann,Krauss] arXiv:0709.1027, [Gehrmann-deRidder,Ritzmann,Skands] arXiv:1108.6172
- Recurring themes, not special to $t\bar{t}b\bar{b}$
  - PS uncertainties hard to judge and reduce
    [Cascioli,Maierhöfer,Moretti,Pozzorini,Siegert] arXiv:1309.591
  - Matching needed for inclusive predictions
    [Krause,Siegert,SH] arXiv:1904.09382, [Ferencz,Katzy,Krause,Pollard,Siegert,SH]

[ATLAS] arXiv:1712.08895



Challenges in modeling HF production

- Both high-energy limit and threshold region should be described well, but
- Infrared finite prediction for $g \to Q\bar{Q}$ leaves splitting functions somewhat arbitrary
- Soft gluon emission off light/heavy quarks associated with $\alpha_s(k_T^2)$, i.e. "correct" scale is k_T^2 [Amati et al.] NPB173(1980)429, but no such argument to set scale for $g \to Q\bar{Q}$
  $\to$ HQ production rate not very stable w.r.t. parton shower variations
- A number of different prescriptions, e.g.
  [Norrbin,Sjöstrand], hep-ph/0010012,
  [Gieseke,Stephens,Webber] hep-ph/0310083,
  [Schumann,Krauss] arXiv:0709.1027,
  [Gehrmann-deRidder,Ritzmann,Skands] arXiv:1108.6172
  varying success in describing expt. data



[Norrbin,Sjöstrand] hep-ph/0010021

- Ideally, $t\bar{t}b\bar{b}$ should be predicted by ME over full phase space

Possible solution: Matching of 4FS and 5FS

- General idea of FONLL: Treatment of logarithms amounts to using two different renormalization schemes
  - 4FS is a decoupling scheme
  - 5FS is a minimal subtraction scheme
- Calculations can be matched by
  - Re-expressing both in same renormalization scheme
  - Subtracting the overlap

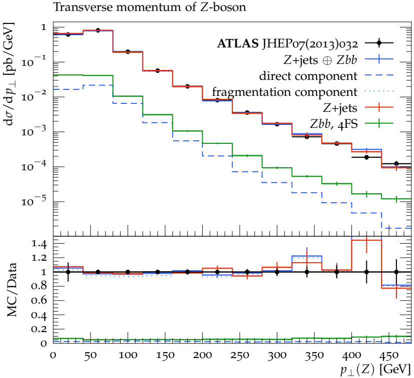
$$\sigma^{\text{FONLL}} = \sigma^{\text{massive}} + (\sigma^{\text{massless}} - \sigma^{\text{massive},\,0})$$

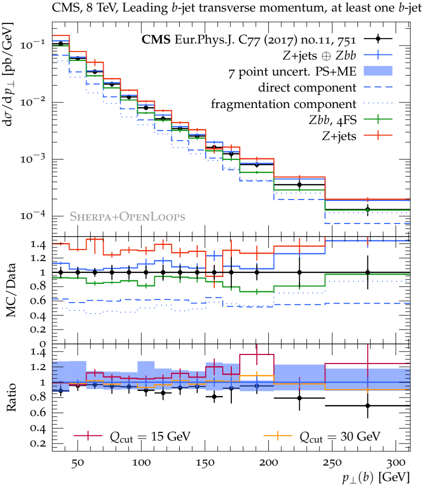
- This has been applied extensively to inclusive observables
  [Cacciari,Frixione,Mangano,Nason,Ridolfi] hep-ph/0312132,
  [Forte,Napoletano,Ubiali] arXiv:1508.01529, arXiv:1607.00389, ...
- Extension to differential observables in MC simulations tested for $V + b\bar{b}$ & $t\bar{t}b\bar{b}$ using “fusing” algorithm
  [Krause,Siegert,SH] arXiv:1904.09382
- Similar MC algorithm can be devised for S-ACOT scheme
  Both can be applied to existing light-flavor event samples

Cross-check: $Z+$jets & $Zb\bar{b}$

- Validation with LHC data

[Krause,Siegert,SH] arXiv:1904.09382





	Data [pb]	**Fusing** [pb]
$Z+\geq 1b$	$3.55 \pm 0.24_{\text{comb}}$	$3.80(5) \pm^{0.83}_{0.33}$
$Z+\geq 2b$	$0.331 \pm 0.037_{\text{comb}}$	$0.282(4) \pm^{0.027}_{0.022}$

Summary and Outlook

- Lots of activity in parton shower development ...
  - Logarithmic precision [PanScales,Deductor,Herwig,Sherpa,...]
  - Higher-order kernels [Vincia,Sherpa,Herwig,...]
  - Interplay w/ NNLL, CMW [PanScales,Sherpa,...]
- ... and matching to fixed-order calculations
  - Improvements at NLO [Herwig,Pythia,Sherpa,...]
  - Resummation based [Geneva,MINNLO$_{PS}$]
  - Fully differential [Vincia,UN^{X}LOPS,TOMTE]
- Still, many questions remain [Campbell et al.] arXiv:2203.11110
  - Systematic treatment of kinematic edge effects
  - Massive quark production & evolution
  - Interplay with hadronization
  - ...

Exciting times ahead!

Backup Slides

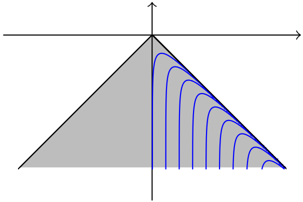
Angular ordered parton showers

- Differential radiation probability

$$\mathrm{d}\mathcal{P} = \mathrm{d}\Phi_{+1}|M|^2 \approx \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2}\,\mathrm{d}z\,\frac{\alpha_s}{2\pi}\,P_{\widetilde{ij}i}(z)$$

  - Ordering parameter $\tilde{q}^2 = \dfrac{2p_ip_j}{z(1-z)} \approx 4E_{\widetilde{ij}}^2 \sin^2\dfrac{\theta_{ij}}{2}$
  - Splitting variable $z = \dfrac{1+\cos\theta_{ik}}{2} = \dfrac{p_ip_k}{(p_i+p_j)p_k}$

- Lund plane filled from center to edges
  - Random walk in p_T^2
  - Color factors correct for observables insensitive to azimuthal correlations
  - Small dead zone at $\ln(p_T^2/\tilde{s}) \approx 0$



- Usually disfavored due to dead zones
  Not suitable to resum non-global logartihms

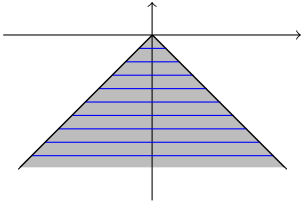
Dipole showers

- Differential radiation probability for the dipole

$$\mathrm{d}\mathcal{P} = \mathrm{d}\Phi_{+1}|M|^2 \approx \frac{\mathrm{d}p_T^2}{p_T^2}\,\mathrm{d}\eta\,\frac{\alpha_s}{2\pi}\,\tilde{P}_{\widetilde{ij}i}(z)$$

  - Ordering parameter p_T^2
  - Splitting variable $z = 1 - \frac{s_{ij}}{s - s_{ij}}\,\mathrm{e}^{-2\eta}$

- Lund plane filled from top to bottom
  - Random walk in η
  - Color factors in CFFE approximation
  - Both ends of dipole evolve simultaneously
  - No dead zones

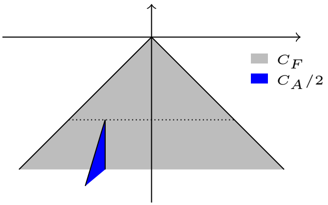


- Solves problem of dead zones
  Known issues with color coherence

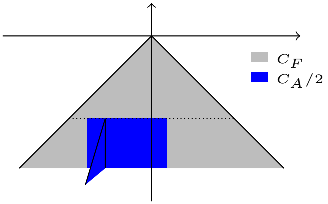
Problems with average color charges

[Gustafsson] NPB392(1993)251

- In angular ordered showers angles are measured in the event center-of-mass frame $\rightarrow$ coherence effects modeled by angular ordering variable agree on average with matrix element



- In dipole-like showers angles effectively measured in center-of-mass frame of emitting color dipole $\rightarrow$ angular coherence not reflected by setting average QCD charge

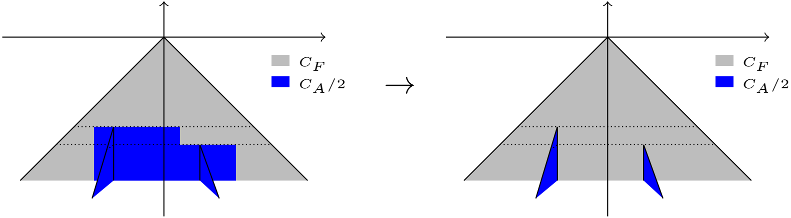


- Emission off "back plane" in Lund diagram should be associated with C_F, but is partly associated with $C_A/2$ in dipole showers
- All-orders problem that appears first in 2-gluon emission case

Solutions for average color charges

[Gustafsson] NPB392(1993)251
[Hamilton,Medves,Salam,Scyboz,Soyez] arXiv:2011.10054

- Analyze rapidity of gluon emission in event center-of-mass frame
- Sectorize phase space and assign gluon to closest parton
  $\to$ choose corresponding color charge for evolution
- Same technology for higher number of emissions

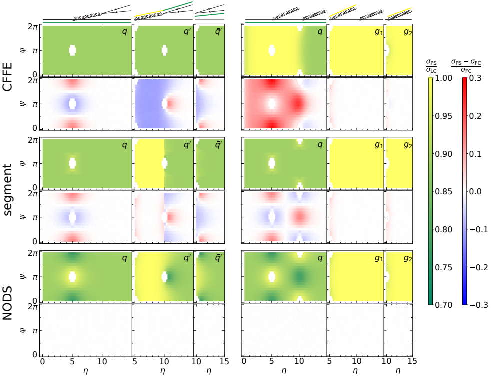


- Starting with 4 emissions, there be “color monsters”
  - Quartic Casimir operators (easy)
  - Non-factorizable contributions (hard)

Solutions for average color charges

[Hamilton,Medves,Salam,Scyboz,Soyez] arXiv:2011.10054

- Can include double-soft corrections via reweighting [Giele,Kosower,Skands] arXiv:1102.2126
- Algorithm scales as N^2 but can be simplified while retaining formal accuracy
- Implementation as nested corrections in rapidity segments of parent dipole
- Excellent agreement with full matrix element



- Good agreement with full-color evolution [Hatta,Ueda] arXiv:1304.6930

Problems with momentum mapping

[Dasgupta,Dreyer,Hamilton,Monni,Salam] arXiv:1805.09327

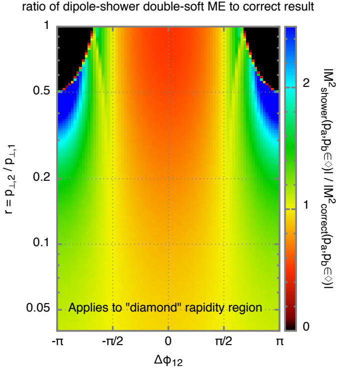
- Subtle problems in standard dipole-like momentum mapping

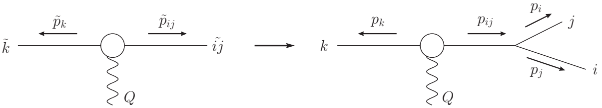
$$p_k^\mu = \left(1 - \frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\right)\tilde{p}_k^\mu$$

$$p_i^\mu = \tilde{z}\,\tilde{p}_{ij}^\mu + (1-\tilde{z})\,\frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\tilde{p}_k^\mu + k_\perp^\mu$$

$$p_j^\mu = (1-\tilde{z})\,\tilde{p}_{ij}^\mu + \tilde{z}\,\frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\tilde{p}_k^\mu - k_\perp^\mu$$

- Induces changes of previously generated soft/collinear momenta
- Spoils agreement w/ analytic resummation





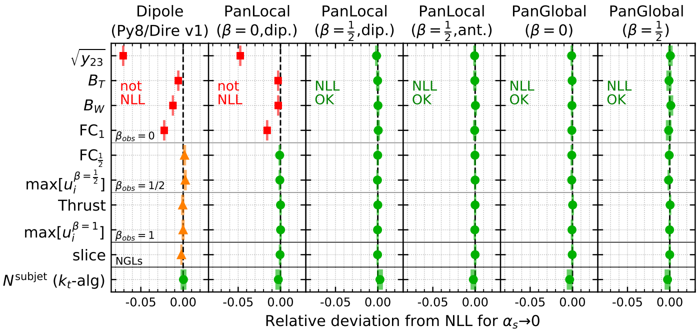
Solutions for momentum mapping

[Dasgupta,Dreyer,Hamilton,Monni,Salam,Soyez] arXiv:2002.11114

- Problem can be solved by partitioning of antenna radiation pattern and choosing a suitable evolution variable ($\beta \sim 1/2$)

$$k_T = \rho v e^{\beta|\bar{\eta}|} \qquad \rho = \left(\frac{s_i s_j}{Q^2 s_{ij}}\right)^{\beta/2}$$

- Different recoil schemes can lead to NLL result if β chosen appropriately: Local dipole, local antenna, and global antenna
- NLL correct for global and non-global observables in $e^+e^- \to$hadrons



Solutions for momentum mapping

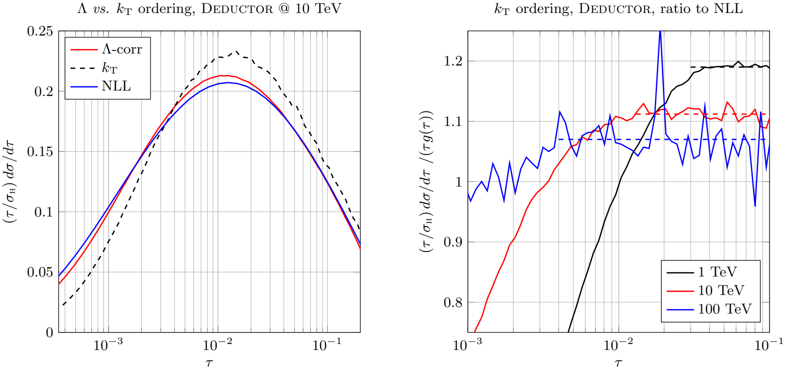
[Bewick,Ferrario-Ravasio,Richardson,Seymour] arXiv:1904.11866, arXiv:2107.04051

- Tradiational Herwig PS angular ordered $\rightarrow$ NLL for global observables
- May be affected by unsuitable, non-traditional kinematics mapping
- In context of angular ordered Herwig 7 (final- and initial-state)
  - q_T preserving scheme:
    Maintains logarithmic accuracy, overpopulates hard region
  - q^2 preserving scheme:
    Breaks logarithmic accuracy, good description of hard region
  - Dot product preserving scheme (new):
    Maintains logarithmic accuracy, good description of hard radiation

Solutions for momentum mapping

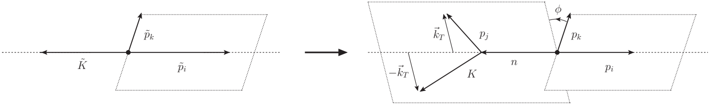
[Nagy,Soper] arXiv:2011.04773

- Local transverse recoil, global longitudinal recoil
- Analytic proof of NLL correctness, based on kinematics in $s \to \infty$ limit



An intuitive solution & analytic proof of correctness

[Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057



- In collinear limit, splitting kinematics defined by ($n \to$ auxiliary vector)

$$p_i \xrightarrow{i||j} z\,\tilde{p}_i\;, \qquad p_j \xrightarrow{i||j} (1-z)\,\tilde{p}_i \qquad \text{where} \qquad z = \frac{p_i n}{(p_i+p_j)n}$$

- Parametrization, using hard momentum $\tilde{K}$

$$p_i = z\,\tilde{p}_i\;, \qquad n = \tilde{K} + (1-z)\,\tilde{p}_i$$

- Using on-shell conditions & momentum conservation ($\kappa = \tilde{K}^2/(2\tilde{p}_i\tilde{K})$)

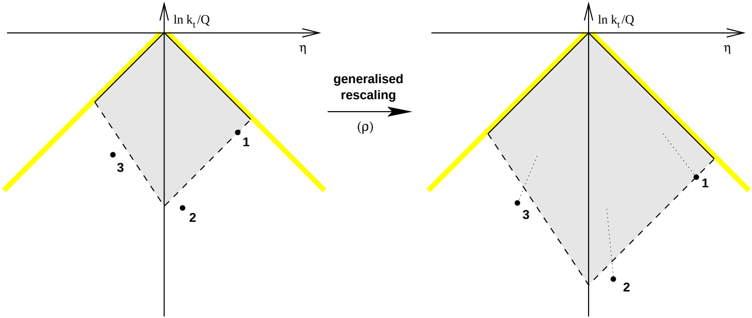
$$\begin{aligned} p_j &= (1-z)\,\tilde{p}_i + v\big(\tilde{K} - (1-z+2\kappa)\,\tilde{p}_i\big) + k_\perp \\ K &= \tilde{K} - v\big(\tilde{K} - (1-z+2\kappa)\,\tilde{p}_i\big) - k_\perp \end{aligned}$$

- Momenta in $\tilde{K}$ Lorentz-boosted to new frame K [Catani,Seymour] hep-ph/9605323

$$p_l^\mu \to \Lambda^\mu{}_\nu(K,\tilde{K})\,p_l^\nu\;, \qquad \Lambda^\mu{}_\nu(K,\tilde{K}) = g^\mu{}_\nu - \frac{2(K+\tilde{K})^\mu(K+\tilde{K})_\nu}{(K+\tilde{K})^2} + \frac{2\tilde{K}^\mu K_\nu}{K^2}\;.$$

An intuitive solution & analytic proof of correctness

[Banfi,Salam,Zanderighi] hep-ph/0407286



- $\alpha_s \to 0$ limit taken by similarity transformation of Lund plane
- Can be parametrized in terms of scaling parameter ρ
  a, b – observable-dependent resummation parameters

$$k_{t,l} \to k'_{t,l} = k_{t,l}\rho^{(1-\xi_l)/a+\xi_l/(a+b)}$$

$$\eta_l \to \eta'_l = \eta - \xi_l \frac{\ln\rho}{a+b}\,, \qquad \text{where} \qquad \xi = \frac{\eta}{\eta_{\max}}$$

- NLL precision requires scaling to be maintained after additional emissions

An intuitive solution & analytic proof of correctness

- Lorentz transformation defined by shift $\tilde{K} \to K$

$$K^\mu = \tilde{K}^\mu - X^\mu \,, \qquad \text{where} \qquad X^\mu = p_j^\mu - (1-z)\,\tilde{p}_i^\mu$$

- X is small, but is it small enough? Rewrite

$$\Lambda^\mu_{\ \nu}(K,\tilde{K}) = g^\mu_\nu + \tilde{K}^\mu A_\nu + X^\mu B_\nu$$

- In NLL limit, coefficients scale as

$$A^\nu \overset{\rho\to 0}{\longrightarrow} 2\,\frac{\tilde{K}X}{\tilde{K}^2}\,\frac{\tilde{K}^\nu}{\tilde{K}^2} - \frac{X^\nu}{\tilde{K}^2}\,, \qquad \text{and} \qquad B^\nu \overset{\rho\to 0}{\longrightarrow} \frac{\tilde{K}^\nu}{\tilde{K}^2}\,.$$

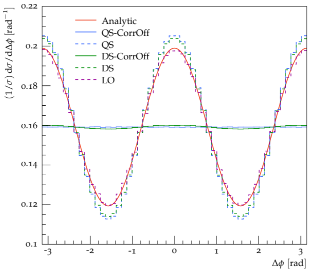
- Simplify situation by taking $a=1$, $b=0$ (worst offenders)
  Relative momentum shift of soft emission particle l becomes

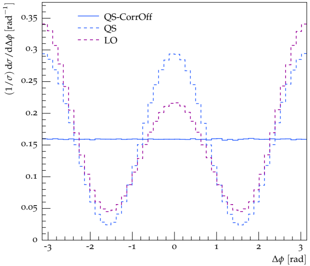
$$\begin{aligned}
\Delta p_l^0/\tilde{p}_l^0 &\sim 2X^0 + \rho^{1-\max(\xi_i,\xi_j)}\tilde{K}^0 &&\sim \rho^{1-\max(\xi_i,\xi_j)} &&\overset{\rho\to 0}{\longrightarrow}\ 0\\
\Delta p_l^3/\tilde{p}_l^3 &\sim X^3 &&\sim \rho^{1-\max(\xi_i,\xi_j)} &&\overset{\rho\to 0}{\longrightarrow}\ 0\\
\Delta p_l^{1,2}/\tilde{p}_l^{1,2} &\sim \rho^{-\xi_l}X^{1,2} &&\sim \rho^{1-\xi_l} &&\overset{\rho\to 0}{\longrightarrow}\ 0
\end{aligned}$$

- **Intuitive picture**: To compensate for recoil, overall multipole boosted
  $\to$ *all* momenta change, recoil effectively distributed among them
  $\to$ change is of order $\mathrm{k}_T/\sqrt{K^2}$ per particle, vanishes as $\mathrm{k}_T \to 0$

Spin correlations

[Richardson,Webster] arXiv:1807.01955



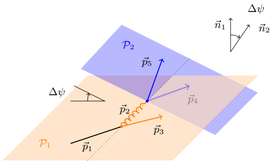


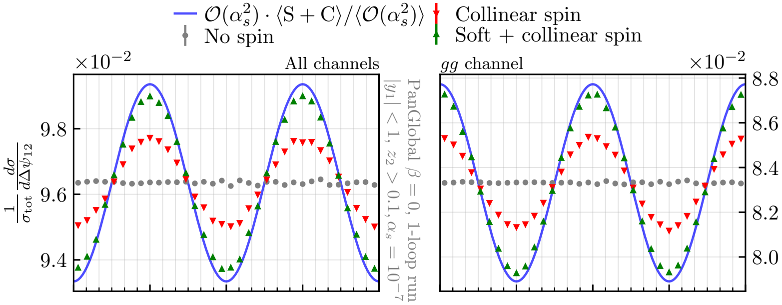
- Azimuthal modulation of QCD radiation due to spin of intermediate gluons
- Linear time algorithm to simulate effect
  [Collins] NPB304(1988)794, [Knowles] CPC58(1990)271, [Richardson] hep-ph/0110108
- Decay correlations in $\sim$ all generators
  [Gigg,Richardson] hep-ph/0703199 [Artoisenet, Frederix,Mattelaer,Rietkerk] arXiv:1212.3460 [Kuttimalai,Schumann,Siegert,SH] arXiv:1412.6478
- Spin-dependent parton showers
  Herwig [Richardson,Webster] arXiv:1807.01955
  (Vincia [Fischer,Lifson,Skands] arXiv:1708.01736)

Spin correlations

[Hamilton,Karlberg,Salam,Scyboz,Verheyen] arXiv:2111.01161

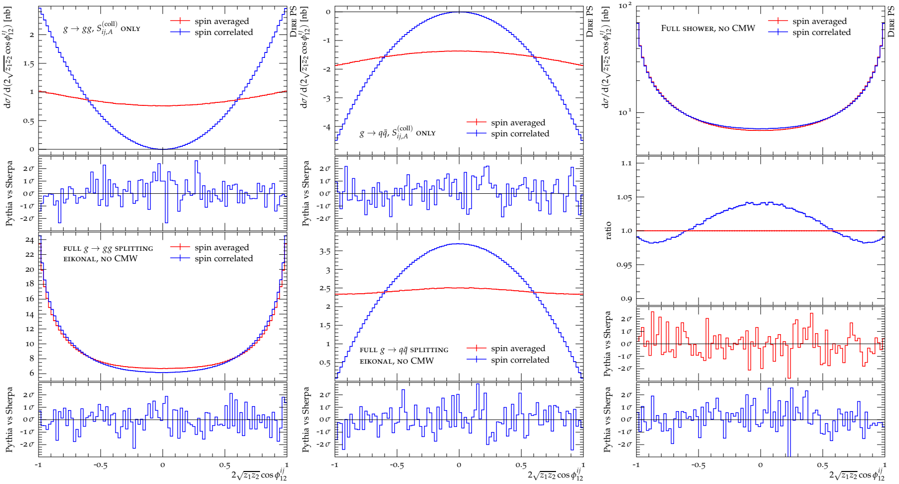
- Azimuthal dependence of radiation pattern due to spinning gluons should be implemented
- Linear time algorithm to simulate effect
  [Collins] NPB304(1988)794, [Knowles] NPB310(1988)571





Spin correlations

[Dulat,Prestel,SH] arXiv:1805.03757



- Spin effects at $\mathcal{O}(\alpha_s^2)$ from double-soft / triple-collinear radiation pattern
- Small overall impact on measurable azimuthal angles at LEP I

Semi-classical radiation pattern

- $U(1)$ point charge on trajectory $y^\mu(s) \to$ conserved current j^μ(x)

$$j^\mu(x) = g\int \mathrm{d}t\,\frac{dy^\mu(t)}{dt}\,\delta^{(4)}(x-y(t))\;, \qquad\qquad g=\sqrt{4\pi\alpha}$$

- Particle receives 'kick' at $t=0 \to$ momentum change $p_k \to p_i$

$$j^\mu(k) = \int \mathrm{d}^4x\, e^{ikx}\, j^\mu(x) = ig\left(\frac{p_k^\mu}{p_k k + i\varepsilon} - \frac{p_i^\mu}{p_i k - i\varepsilon}\right)$$

- Probability of single-gluon final state sourced by $j^\mu(k)$ at $\mathcal{O}(\alpha_s)$

$$\begin{aligned}\int \mathrm{d}W^{2\,(1)}_{a\to bc}(p_j) &= \int \frac{\mathrm{d}^3\vec{p}_j}{(2\pi)^3\,2E_j}\left|\,i\int \mathrm{d}^4x\, j^\mu(x)\langle\vec{p}_j|A_\mu(x)|0\rangle\right|^2\\ &= -\int \frac{\mathrm{d}^3\vec{p}_j}{(2\pi)^3\,2E_j}\sum_{\lambda=\pm}\big(j(p_j)\varepsilon_\lambda(p_j)\big)\big(j(p_j)\varepsilon_\lambda(p_j)\big)^*\\ &\to |g|^2\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^\varepsilon \int \frac{\mathrm{d}^D\vec{p}_j}{(2\pi)^D}\,\frac{2p_ip_k}{(p_ip_j)(p_kp_j)}\,\delta(p_j^2)\end{aligned}$$

  - Universal, semi-classical integrand
  - Originates in gauge boson radiation off conserved charge
  - Leads to double logarithm $1/2\log^2(2p_ip_k/\mu^2)$

Semi-classical radiation pattern

[Marchesini,Webber] NPB310(1988)461

- Soft gluon radiator can be written in terms of energies and angles

$$J_\mu J^\mu \to \frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{W_{ik,j}}{E_j^2}$$

Angular "radiator" function

$$W_{ik,j} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})}$$

- Divergent as $\theta_{ij} \to 0$ and as $\theta_{jk} \to 0$

$\to$ Expose individual collinear singularities using $W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j}$

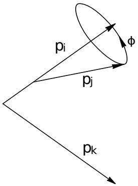
$$\tilde{W}^i_{ik,j} = \frac{1}{2}\left[\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{kj})} + \frac{1}{1-\cos\theta_{ij}} - \frac{1}{1-\cos\theta_{kj}}\right]$$

  - Divergent as $\theta_{ij} \to 0$, but regular as $\theta_{kj} \to 0$
  - Convenient properties upon integration over azimuthal angle

Semi-classical radiation pattern

- Work in a frame where direction of $\vec{p}_i$ aligned with z-axis

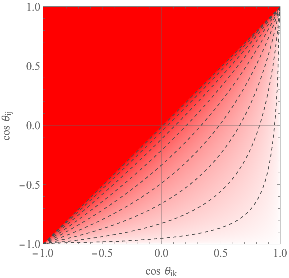
$$\cos\theta_{kj} = \cos\theta_k^i \cos\theta_j^i + \sin\theta_k^i \sin\theta_j^i \cos\phi_{kj}^i$$

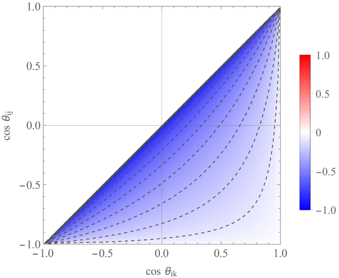


- Integration over ϕ_j yields

$$\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi_{kj}^i \tilde{W}_{ik,j}^i = \frac{1}{1-\cos\theta_j^i} \times \begin{cases} 1 & \text{if} \quad \theta_j^i < \theta_k^i \\ 0 & \text{else} \end{cases}$$

  - On average, no radiation outside cone defined by parent dipole
  - Differential radiation pattern more intricate:
    Positive & negative contributions outside cone sum to zero

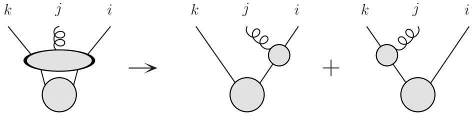




Semi-classical radiation pattern

- Alternative to angular ordering: partial fraction matrix element & match to collinear sectors [Ellis,Ross,Terrano] NPB178(1981)421, [Catani,Seymour] hep-ph/9605323

$$\frac{W_{ik,j}}{E_j^2} \rightarrow \frac{1}{p_i p_j}\frac{p_i p_k}{(p_i+p_k)p_j} + \frac{1}{p_k p_j}\frac{p_i p_k}{(p_i+p_k)p_j}$$



  - Captures matrix element both in angular ordered and unordered region
  - Caveat: Oversampling difficult for certain kinematics maps

- Separate into energy & angle first [Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057
  Partial fraction angular radiator only: $W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j}$

$$\bar{W}^i_{ik,j} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(2-\cos\theta_{ij}-\cos\theta_{kj})}$$

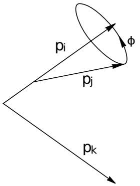
  - Bounded by $(1-\cos\theta_{ij})\bar{W}^i_{ik,j} < 2$
  - Strictly positive

Semi-classical radiation pattern

- Integration over ϕ_j yields

$$\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi^{i}_{kj}\,\bar{W}^{i}_{ik,j} = \frac{1}{1-\cos\theta^{i}_{j}}\,\frac{1}{\sqrt{(\bar{A}^{i}_{ij,k})^2-(\bar{B}^{i}_{ij,k})^2}}$$

  - Radiation across all of phase space
  - Probabilistic radiation pattern



$$\bar{A}^{i}_{ij,k} = \frac{2-\cos\theta^{i}_{j}(1+\cos\theta^{i}_{k})}{1-\cos\theta^{i}_{k}}$$

$$\bar{B}^{i}_{ij,k} = \frac{\sqrt{(1-\cos^2\theta^{i}_{j})(1-\cos^2\theta^{i}_{k})}}{1-\cos\theta^{i}_{k}}$$