Mc@Nlo vs Powheg   Opportunities and Limitations

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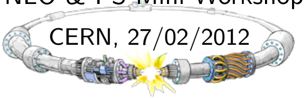
arXiv:1111.1220 [hep-ph], arXiv:1201.5882 [hep-ph]

NLO & PS Mini Workshop



CERN, 27/02/2012

Basics of POWHEG

Assume parton shower (PS) with same structure as NLO-subtraction method

Expectation value of observable O to $\mathcal{O}(\alpha_s)$ in parton-shower approximation:

$$\langle O \rangle = \sum \int \mathrm{d}\Phi_B \, \mathrm{B} \left[\Delta^{(\mathrm{PS})}(t_0) \, O(\Phi_B) + \sum \int_{t_0} \mathrm{d}\Phi^{ij,k}_{R|B} \, \mathrm{K}_{ij,k} \, \Delta^{(\mathrm{PS})}(t(\Phi_{R|B})) \, O(\Phi_R) \right]$$

where $\Delta^{(\mathrm{PS})}(t) = \exp\left\{ -\int_t \mathrm{d}\Phi^{ij,k}_{R|B} \, \mathrm{K}_{ij,k} \right\}$

Make this NLO-correct:

- Radiation pattern of R from ME correction ($\mathrm{D}^{(\mathrm{S})}_{ij,k} \to$ subtraction term)

$$w = \mathrm{R}_{ij,k} \, / \, \mathrm{B}\mathrm{K}_{ij,k}, \quad \text{where} \quad \mathrm{R}_{ij,k} = \rho_{ij,k} \mathrm{R} \quad \text{and} \quad \rho_{ij,k} = \mathrm{D}^{(\mathrm{S})}_{ij,k} \, / \sum \mathrm{D}^{(\mathrm{S})}_{mn,l}$$

- Replace "seed cross section" by ($\mathrm{I}^{(\mathrm{S})} \to$ integrated subtraction terms)

$$\bar{\mathrm{B}} = \mathrm{B} + \tilde{\mathrm{V}} + \mathrm{I}^{(\mathrm{S})} + \sum \int \mathrm{d}\Phi^{ij,k}_{R|B} \left[\mathrm{R}_{ij,k} - \mathrm{D}^{(\mathrm{S})}_{ij,k} \right]$$

Combine ME-correction and local K-factor $\to$ POWHEG

[Nason] JHEP11(2004)040 [Frixione,Nason,Oleari] JHEP11(2007)070

$$\langle O \rangle = \sum \int \mathrm{d}\Phi_B \, \bar{\mathrm{B}} \left[\bar{\Delta}^{(\mathrm{R}/\mathrm{B})}(t_0) \, O(\Phi_B) + \sum \int_{t_0} \mathrm{d}\Phi^{ij,k}_{R|B} \, \frac{\mathrm{R}_{ij,k}}{\mathrm{B}} \, \bar{\Delta}^{(\mathrm{R}/\mathrm{B})}(t(\Phi_{R|B})) \, O(\Phi_R) \right]$$

POWHEG as implemented in SHERPA

Aimed for:

- Automated, process-independent implementation
- Use of existing code for Catani-Seymour dipole subtraction
  [Gleisberg,Krauss] EPJC53(2008)501
- Use of existing dipole-like parton shower
  [Krauss,Schumann] JHEP03(2008)038, [Schumann,SH,FS] PRD81(2010)034026

Lessons learned in next-to-simplest scenario (W/Z+1-jet):

- Using dipole subtraction to compute $\bar{\mathrm{B}}$ is harder than expected
  numerical instabilities due to cuts on underlying Born process, which have to be applied separately for every dipole term
- Parton shower would have to be "dipole-term corrected" first
  NLO-accuracy depends crucially on exact same dipole terms in both the subtracted matrix element and the parton shower

Could have changed to different subtraction scheme, but decided to press on with solution for CS method

→ **MC@NLO came to the rescue**

Parton-shower perspective → only "soft" part $\mathrm{D}^{(\mathrm{A})}_{ij,k}$ of $\mathrm{R}_{ij,k}$ exponentiated

Defines MC@NLO algorithm [Frixione,Webber] JHEP06(2002)029

$$\langle O\rangle = \sum\int \mathrm{d}\Phi_B\,\bar{\mathrm{B}}^{(\mathrm{A})}\left[\bar{\Delta}^{(\mathrm{A})}(t_0)\,O(\Phi_B) + \sum\int_{t_0}\mathrm{d}\Phi^{ij,k}_{R|B}\,\frac{\mathrm{D}^{(\mathrm{A})}_{ij,k}}{\mathrm{B}}\,\bar{\Delta}^{(\mathrm{A})}(t(\Phi_{R|B}))\,O(\Phi_R)\right] + \sum\int \mathrm{d}\Phi_R\left[\mathrm{R}_{ij,k} - \mathrm{D}^{(\mathrm{A})}_{ij,k}\right]O(\Phi_R)$$

Seed cross sections and Sudakov form factors change accordingly:

$$\bar{\mathrm{B}}^{(\mathrm{A})} = \mathrm{B} + \tilde{\mathrm{V}} + \mathrm{I}^{(\mathrm{S})} + \sum\int \mathrm{d}\Phi^{ij,k}_{R|B}\left[\mathrm{D}^{(\mathrm{A})}_{ij,k} - \mathrm{D}^{(\mathrm{S})}_{ij,k}\right]$$

Note that $\bar{\Delta}^{(\mathrm{A})} \neq \Delta^{(\mathrm{PS})}$, as soft-gluon limit not exact in PS

Plain POWHEG recovered as special case of MC@NLO $(\mathrm{D}^{(\mathrm{A})}_{ij,k} \to \mathrm{R}_{ij,k})$

Substantial simplification if $\mathrm{D}^{(\mathrm{A})}_{ij,k} \to \mathrm{D}^{(\mathrm{S})}_{ij,k}$ ⇒ integral in $\bar{\mathrm{B}}^{(\mathrm{A})}$ can be dropped

Note:

- Varying $\mathrm{D}^{(\mathrm{A})}_{ij,k}$ changes properties of resummation
  [Alioli,Nason,Oleari,Re] JHEP04(2009)002, [SH,FK,MS,FS] arXiv:1111.1220
- $\mathrm{D}^{(\mathrm{A})}_{ij,k}$ may differ from $\mathrm{D}^{(\mathrm{S})}_{ij,k}$ or $\mathrm{R}_{ij,k}$ by simple cuts
  Used to implement resummation scale Q^2 (upper scale of PS evolution)

MC@NLO in the $\mathrm{D}^{(\mathrm{A})} = \mathrm{D}^{(\mathrm{S})}$-scheme

SHERPA implements simplified MC@NLO by “dipole-term correction”

Tricky point: negative weights due to $\mathrm{D}^{(\mathrm{A})} < 0$ e.g. subleading color dipoles

Use modified Sudakov veto algorithm to correct [SH,FK,MS,FS] arXiv:1111.1220

- Assume $f(t)$ as function to be generated, and overestimate $g(t)$
  Standard probability for one acceptance with n rejections

$$\frac{f(t)}{g(t)}\,g(t)\,\exp\left\{-\int_t^{t_1}\mathrm{d}\bar{t}\,g(\bar{t})\right\}\prod_{i=1}^{n}\left[\int_{t_{i-1}}^{t_{n+1}}\mathrm{d}t_i\left(1-\frac{f(t_i)}{g(t_i)}\right)g(t_i)\,\exp\left\{-\int_{t_i}^{t_{i+1}}\mathrm{d}\bar{t}\,g(\bar{t})\right\}\right]$$

- Can split weight into MC and analytic part using auxiliary function $h(t)$

$$\frac{f(t)}{g(t)}\,h(t)\,\exp\left\{-\int_t^{t_1}\mathrm{d}\bar{t}\,h(\bar{t})\right\}\prod_{i=1}^{n}\left[\int_{t_{i-1}}^{t_{n+1}}\mathrm{d}t_i\left(1-\frac{f(t_i)}{g(t_i)}\right)h(t_i)\,\exp\left\{-\int_{t_i}^{t_{i+1}}\mathrm{d}\bar{t}\,h(\bar{t})\right\}\right]$$

$$w(t,t_1,\ldots,t_n) = \frac{g(t)}{h(t)}\prod_{i=1}^{n}\frac{g(t_i)}{h(t_i)}\,\frac{h(t_i)-f(t_i)}{g(t_i)-f(t_i)}$$

Identify $f(t)$, $g(t)$ and $h(t)$:

- $f(t)$ determined by MC@NLO $\rightarrow \mathrm{D}^{(\mathrm{A})}$
- $h(t)$ determined by PS $\rightarrow \mathrm{D}^{(\mathrm{PS})}$
- **g(t) can be chosen freely $\rightarrow$ const $\cdot$ f**
  constraints: $\mathrm{sign}(f) = \mathrm{sign}(g)$, $|f| \leq |g|$

MC@NLO as implemented in SHERPA

Initial expectations for POWHEG finally met by MC@NLO:

- Easy to automate and process-independent
  Only finite piece V of virtual correction to be supplied
- Based on Catani-Seymour dipole subtraction
- Using existing dipole-like parton shower

Lessons learned in next-to-simplest scenario (W/Z+1-jet):

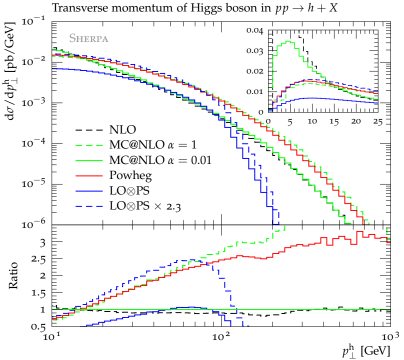
- Computing $\bar{\mathrm{B}}^{(\mathrm{A})}$ with dipole subtraction is simpler than we thought
  no numerical instabilities as we do not project R onto $\mathrm{R}_{ij,k}$ for $\bar{\mathrm{B}}^{(\mathrm{A})}$
- Parton showers are easy to correct with matrix-elements
  Ratio $\mathrm{D}_{ij,k}^{(\mathrm{A})}/\mathrm{B}$ always non-zero and close to parton-shower result
  Deviations in soft-gluon regime limited by shower cutoff

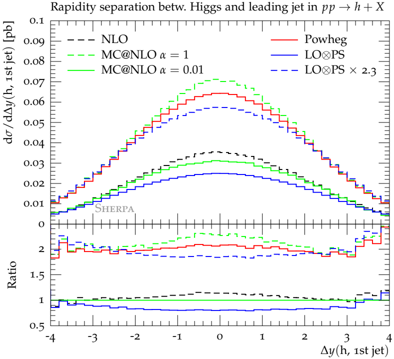
MC@NLO in $\mathrm{D}^{(\mathrm{A})} = \mathrm{D}^{(\mathrm{S})}$-scheme profits from known analytic integrals in $\mathrm{I}^{(\mathrm{S})}$
Sherpa variant inherits excellent phase-space mapping from CS dipole terms

Matrix-element corrections to the parton shower are cheap and easy to evaluate
All input is tree-level like and automatically provided with NLO calculations

Origin of differences between POWHEG and MC@NLO

Q^2 in SHERPA MC@NLO currently set by α_{cut} [Nagy] PRD68(2003)094002





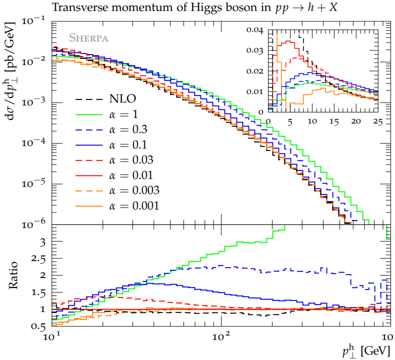
Sanity checks and cross-checks passed:

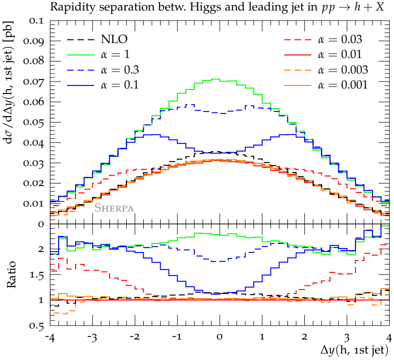
- Sudakov shape of LO⊗PS result reproduced for $\alpha_{\text{cut}} \to 1$
- High-p_T tail of NLO result reproduced for $\alpha_{\text{cut}} \ll 1$

Ideally could do both at the same time (original MC@NLO code does)
Currently limited by inappropriate choice of Q^2, fixed through α_{cut}
To be improved with new phase-space limits on $\mathrm{D}^{(S)}$ → Marek's talk

Origin of differences between POWHEG and MC@NLO

Even though α_{cut} is sub-optimal, it povides a handle for varying Q^2





Essential features of POWHEG analysis JHEP04(2009)002 **reproduced**:

- Hardness of POWHEG p_T spectra for $\alpha_{\text{cut}} \to 1$
- Dip in MC@NLO Δy-spectra for intermediate α_{cut}

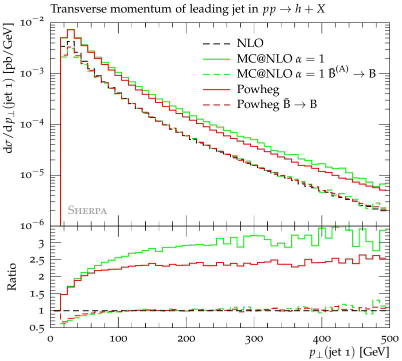
Should not play this game and rather define $\mathrm{D}^{(A)}$ properly ⚠

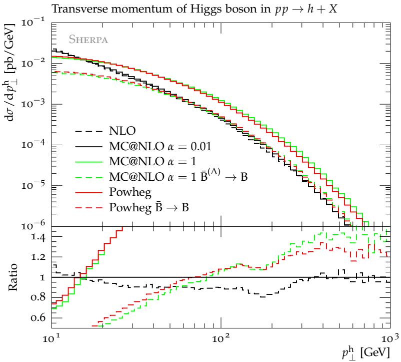
Nevertheless interesting to see that all known effects can be mimicked

Origin of differences between POWHEG and MC@NLO

POWHEG ↔ MC@NLO analyzed for $gg \to h$ in JHEP04(2009)002

Differences attributed to shift from B to $\bar{\mathrm{B}}$. Check this carefully:





Linear plot and large diff range on the left easily fool the eye

Difference POWHEG($\bar{\mathrm{B}} \to \mathrm{B}$) ↔ NLO still ≥20% at large $p_{T,h}$

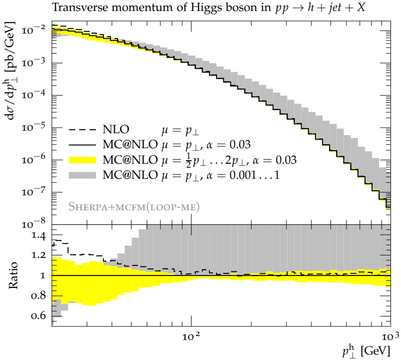
Same for MC@NLO($\bar{\mathrm{B}}^{(\mathrm{A})} \to \mathrm{B}$) if $\alpha \to 1$, but *no difference* if $\alpha \ll 1$

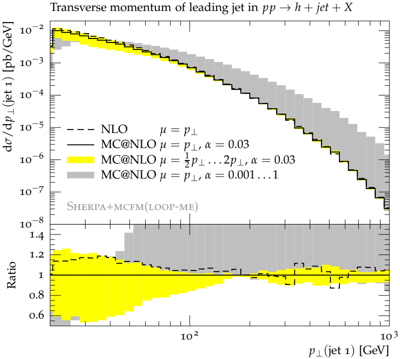
True discrepancy *not* from $\mathrm{B} \to \bar{\mathrm{B}}$, but from $Q^2 \gg m_h$ ⚠

Was hinted at in JHEP04(2009)002 itself → suppression factor proposed

Increased parton multiplicity worsens problems
more QCD partons radiate more → higher chance to go wrong

Exemplified this in the process $pp \to h{+}j$





Interpret this as POWHEG result with varying singular piece of $\mathrm{R}_{ij,k}$

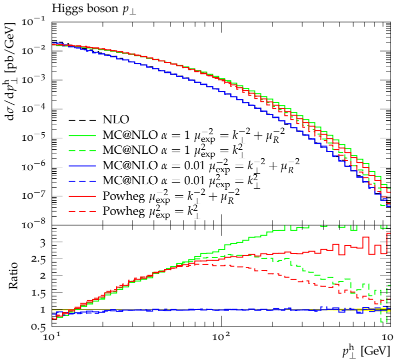
Gray band due to beyond-NLL effects? [Nason,Ridolfi] JHEP08(2006)077
Unlikely, as Q^2 should really be $\mathcal{O}(m_h)$ to avoid large spurious logs
[Banfi,Salam,Zanderighi] JHEP08(2004)062, [Bozzi,Catani,DeFlorian,Grazzini] NPB737(2006)73

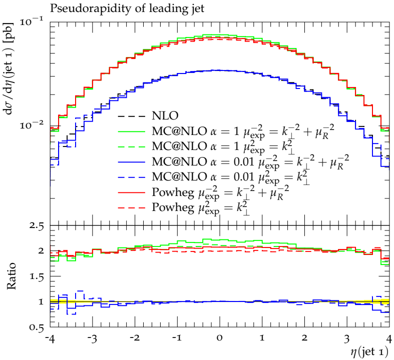
Choice of scales

Choice of scale μ_{exp} in $\bar{\Delta}^{(\text{A})}$ largely arbitrary

Must reduce to transverse momentum in soft/collinear regime

Numerical effects of varying μ_{exp} rather large:





POWHEG & MC@NLO formulae rely on first-order expansion of $\bar{\Delta}^{(\text{A})}$

Formal NLO-accuracy for all scales, but $Q^2 \gg m_h^2$ induces large spurious logs

Exemplifies once more importance of proper Q^2 ⚠

Is complexity a hidden issue?

MC@NLO shown to work well in $W+n$ jets, where $n \leq 3$ at present

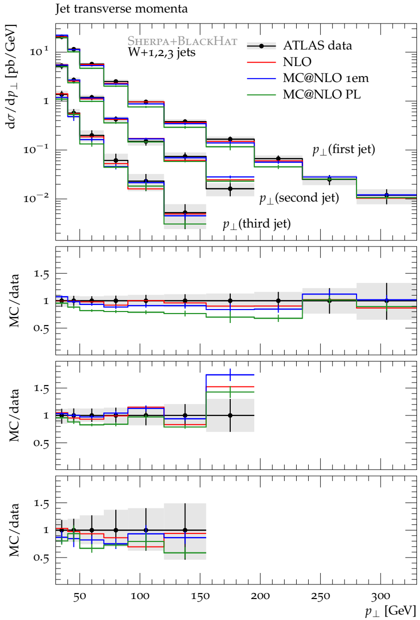
[Frederix,Frixione,Hirschi,Maltoni,Pittau,Torrielli] arXiv:1110.5502 [SH,FK,MS,FS] arXiv:1201.5882

Most general color structures already present in $W+3$ jets

Any more complications unlikely

... but keep your fingers crossed !

Probably fair to say that current bottleneck in MC is *not* matching of NLO & PS at fixed multiplicity



- MC@NLO $\mathrm{D}^{(\mathrm{A})} = \mathrm{D}^{(\mathrm{S})}$-scheme first implemented
- No conceptual or practical obstacles for high-multiplicity processes
- Independent check of POWHEG $\leftrightarrow$ MC@NLO discrepancies

Open to discussions