

# Dire status report

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Parton Showers and Resummation 2017  
Cambridge, 03/28/2017



## Basic layout of Dire [Prestel,SH] arXiv:1506.05057

- ▶ Dipole-like parton shower, kernels as close as possible to DGLAP
- ▶ Partial fraction soft eikonal à la Catani-Seymour, evolve in dipole- $k_T$
- ▶ Two independent implementations (Pythia & Sherpa)
- ▶ Cross-validation at particle level

## New developments [Prestel,SH], [Krauss,Prestel,SH] any time soon

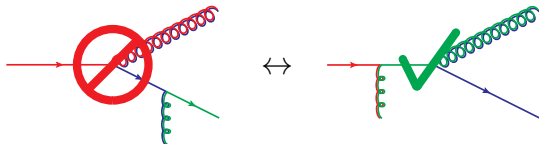
- ▶ MC counterterms implemented in Amegic & Comix
- ▶ MC@NLO matching & NLO subtraction available in Sherpa
- ▶ UNLOPS / MEPS@NLO merging being developed in Pythia / Sherpa
- ▶ 3-loop cusp term and integrated NLO DGLAP kernels included<sup>1</sup>
- ▶ Flavor-changing triple collinear splitting functions included

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<sup>1</sup>Working on corrections to account for choice of dipole- $k_T$  as evolution variable

[Marchesini,Webber] NPB310(1988)461

- ▶ Individual color charges inside a color dipole cannot be resolved by gluons of wavelength larger than the dipole size  
→ emission off combined mother parton instead



- ▶ Net effect is destructive interference outside cone with opening angle defined by emitting color dipole  
→ Soft anomalous dimension halved due to reduced phase space
- ▶ Formerly implemented by angular ordering / angular veto  
[Webber at al.] hep-ph/0210213, [Sjöstrand et al.] hep-ph/0603175
- ▶ Alternative description in terms of color dipoles  
[Gustafsson,Pettersson] NPB306(1988)746, [Kharraziha,Lönnblad] hep-ph/9709424  
[Winter,Krauss] arXiv:0712.3913

- ▶ Angular ordered / vetoed parton shower does not fill full phase space  
Dipole shower lacks parton interpretation  $\rightarrow$  prefer alternative to both
- ▶ Can preserve parton picture by partial fractioning soft eikonal  
 $\leftrightarrow$  soft enhanced part of splitting function [Catani,Seymour] hep-ph/9605323

$$\frac{p_i p_k}{(p_i p_j)(p_j p_k)} \rightarrow \frac{1}{p_i p_j} \frac{p_i p_k}{(p_i + p_k) p_j} + \frac{1}{p_k p_j} \frac{p_i p_k}{(p_i + p_k) p_j}$$

- ▶ “Spectator”-dependent kernels, singular in soft-collinear region only  
 $\rightarrow$  capture dominant coherence effects (3-parton correlations)

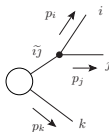
$$\frac{1}{1-z} \rightarrow \frac{1-z}{(1-z)^2 + \kappa^2} \quad \kappa^2 = \frac{k_{\perp}^2}{Q^2}$$

- ▶ For correct soft evolution, ordering variable must be identical at both “dipole ends” ( $\rightarrow$  recover soft eikonal at integrand level)

# The midpoint between dipole and parton showers

Choose parametrization such that soft term is  $\frac{1-z}{(1-z)^2 + \kappa^2}$  in all dipole types

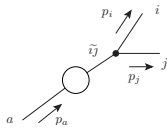
(1) FF



$$\kappa^2 = \frac{p_i p_j p_j p_k}{(p_{\tilde{ij}} p_{\tilde{k}})^2}$$

$$z_j = \frac{p_j p_k}{p_{\tilde{ij}} p_{\tilde{k}}}$$

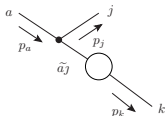
(2) FI



$$\kappa^2 = \frac{p_i p_j p_j p_a}{(p_{ij} p_a)^2}$$

$$z_j = \frac{p_j p_a}{p_{ij} p_a}$$

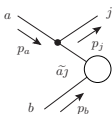
(3) IF



$$\kappa^2 = \frac{p_a p_j p_j p_k}{(p_{jk} p_a)^2}$$

$$z_j = \frac{p_j p_k}{p_{jk} p_a}$$

(4) II



$$\kappa^2 = \frac{p_a p_j p_j p_b}{(p_a p_b)^2}$$

$$z_j = \frac{p_j p_b}{p_a p_b}$$

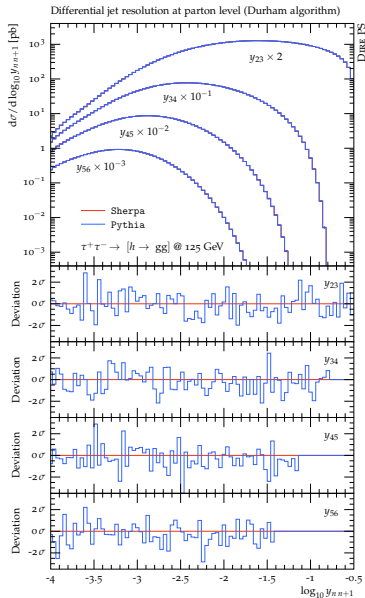
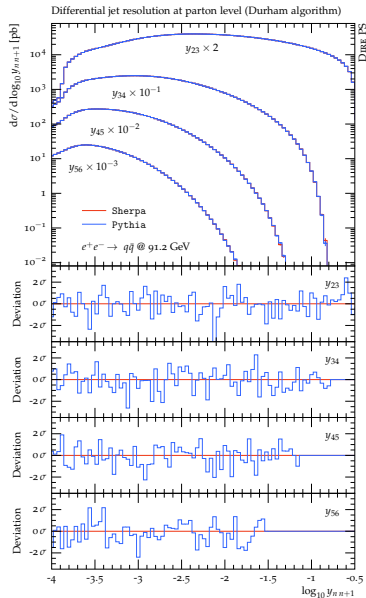
Preserve collinear anomalous dimensions & sum rules  $\rightarrow$  splitting functions fixed

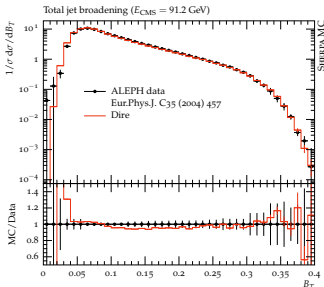
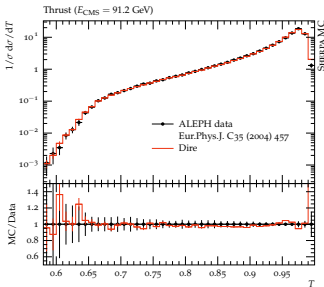
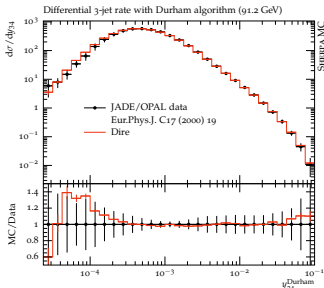
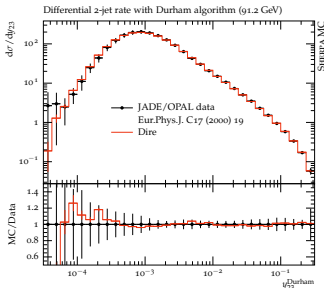
$$P_{qq}(z, \kappa^2) = 2 C_F \left[ \left( \frac{1-z}{(1-z)^2 + \kappa^2} \right)_+ - \frac{1+z}{2} \right] + \gamma_q \delta(1-z)$$

$$P_{gg}(z, \kappa^2) = 2 C_A \left[ \left( \frac{1-z}{(1-z)^2 + \kappa^2} \right)_+ + \frac{z}{z^2 + \kappa^2} - 2 + z(1-z) \right] + \gamma_g \delta(1-z)$$

$$P_{qg}(z, \kappa^2) = 2 C_F \left[ \frac{z}{z^2 + \kappa^2} - \frac{2-z}{2} \right] \quad P_{gq}(z, \kappa^2) = T_R \left[ z^2 + (1-z)^2 \right]$$

# Validation in $e^+e^- \rightarrow \text{hadrons}$





- ▶ NLO DGLAP kernels known since long

- [Curci,Furmanski,Petronzio] NPB175(1980)27, PLB97(1980)437
  - [Floratos,Kounnas,Lacaze] NPB192(1981)417

- ▶ Incorrect scheme for parton showers evolving in  $k_T$ , but
    - ▶ Correction terms straightforward (↗ later)

- ▶ Focus on MC implementation technology first

- ▶ Subtract 2-loop cusp term from NLO kernels and combine with LO soft (CMW rescaling [Catani,Marichesini,Webber] NPB349(1991)635)
  - Include 3-loop term in the same way [Moch,Vermaseren,Vogt] hep-ph/0403192
  - ▶ Redefine time-like Sudakovs to recover NLO DGLAP evolution [Jadach,Skrzypek] hep-ph/0312355
  - ▶ Mostly negative NLO corrections require weighted veto algorithm [Schumann,Siegert,SH] arXiv:0912.3501, [Lönnblad] arXiv:1211.7204
  - ▶ Flavor changing splitting functions require  $2 \rightarrow 4$  transitions [Prestel,SH] any time soon

- ▶ Unsurprisingly the  $2 \rightarrow 4$  technology took longest to develop



- DGLAP equation for fragmentation functions

$$\frac{dx D_a(x, t)}{d \ln t} = \sum_{b=q,g} \int_0^1 d\tau \int_0^1 dz \frac{\alpha_s}{2\pi} [zP_{ab}(z)]_+ \tau D_b(\tau, t) \delta(x - \tau z)$$

- Define plus prescription  $[zP_{ab}(z)]_+ = \lim_{\varepsilon \rightarrow 0} zP_{ab}(z, \varepsilon)$

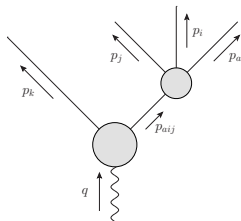
$$P_{ab}(z, \varepsilon) = P_{ab}(z) \Theta(1 - z - \varepsilon) - \delta_{ab} \sum_{c \in \{q,g\}} \frac{\Theta(z - 1 + \varepsilon)}{\varepsilon} \int_0^{1-\varepsilon} d\zeta \zeta P_{ac}(\zeta)$$

- Rewrite for finite  $\varepsilon$

$$\begin{aligned} \frac{1}{D_a(x, t)} \frac{dD_a(x, t)}{d \ln t} = & - \sum_{c=q,g} \int_0^{1-\varepsilon} d\zeta \zeta \frac{\alpha_s}{2\pi} P_{ac}(\zeta) \\ & + \sum_{b=q,g} \int_x^{1-\varepsilon} \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{ab}(z) \frac{D_b(x/z, t)}{D_a(x, t)} \end{aligned}$$

- First term is derivative of Sudakov  $\rightarrow$  beyond LO forward Sudakovs must be computed with symmetry factors replaced by  $z$  [Jadach, Skrzypek] hep-ph/0312355  
Otherwise collinear anomalous dimensions will diverge

- ▶ New topology at NLO from  $q \rightarrow \bar{q}$  and  $q \rightarrow q'$  splittings
- ▶ Generic  $1 \rightarrow 3$  process in parton shower  
 $2 \rightarrow 4$  process in dipole(-like) shower
- ▶ First branching treated as soft gluon radiation, second as collinear splitting (to match diagrammatic structure)
- ▶ Requires new kinematics and suitable phase-space parametrization, set up similar to [Dittmaier] hep-ph/9904440



- Define evolution & splitting variables

$$t = \frac{4 p_j p_{ai} p_{ai} p_k}{q^2 - m_{aij}^2 - m_k^2}, \quad z_a = \frac{2 p_a p_k}{q^2 - m_{aij}^2 - m_k^2}$$

$$s_{ai} = 2 p_a p_i + m_a^2 + m_i^2, \quad x_a = \frac{p_a p_k}{p_{ai} p_k}$$

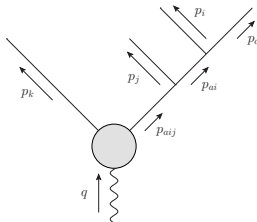
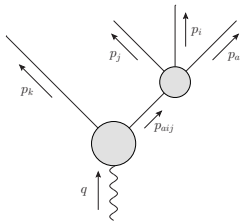
- First branching  $(\tilde{a}i\tilde{j}, \tilde{k}) \rightarrow (ai, j, k)$  constructed with  $m_{ai}^2 \rightarrow s_{ai}$ , using [Catani, Dittmaier, Seymour, Trocsanyi] hep-ph/0201036 [Prestel, SH] arXiv:1506.05057

$$y = \frac{t x_a / z_a}{q^2 - s_{ai} - m_j^2 - m_k^2}$$

$$\tilde{z} = \frac{z_a / x_a}{1 - y} \frac{q^2 - m_{aij}^2 - m_k^2}{q^2 - s_{ai} - m_j^2 - m_k^2}.$$

- Second step now a decay  $(ai, k) \rightarrow (a, i, k)$  can use CDST algorithm with

$$y' = \left[ 1 + \frac{z_a}{x_a} \frac{q^2 - m_{aij}^2 - m_k^2}{s_{ai} - m_a^2 - m_i^2} \right]^{-1}, \quad \tilde{z}' = x_a$$



- Phase space factorization derived similar to [Dittmaier] hep-ph/9904440  
→ s-channel factorization over  $p_{aij}$ , subsequently over  $p_{ai}$

$$\begin{aligned} \int d\Phi(p_a, p_i, p_j, p_k | q) &= \int \frac{ds_{aij}}{2\pi} \int d\Phi(p_{aij}, p_k | q) \int d\Phi(p_a, p_i, p_j | p_{aij}) \\ &= \int d\Phi(\tilde{p}_{aij}, \tilde{p}_k | q) \int [d\Phi(p_a, p_i, p_j | \tilde{p}_{aij}, \tilde{p}_k)] \end{aligned}$$

- Obtain almost fully factorized form

$$\begin{aligned} \int [d\Phi(p_a, p_i, p_j | \tilde{p}_{aij}, \tilde{p}_k)] &= \\ \left[ \frac{J_{\text{FF}}^{(1)}}{4(2\pi)^3} \int \frac{dt}{t} \int dz_a \int d\phi_j \right] &\left[ \frac{1}{4(2\pi)^3} \int ds_{ai} \int \frac{dx_a}{x_a} \int d\phi_i J_{\text{FF}}^{(2)} \right] 2 p_{ai} p_j \end{aligned}$$

- Simple Jacobians that reduce to unity for massless partons

$$J_{\text{FF}}^{(1)} = \frac{q^2 - m_{aij}^2 - m_k^2}{\sqrt{\lambda(q^2, m_{aij}^2, m_k^2)}}, \quad J_{\text{FF}}^{(2)} = \frac{s_{aik} - s_{ai} - m_k^2}{\sqrt{\lambda(s_{aik}, s_{ai}, m_k^2)}}.$$

- ▶ Combination with massless matrix element in collinear limit leads to

$$\begin{aligned} & \int d\Phi(p_a, p_i, p_j, p_k | q) |M_{n+2}(a, i, j, k | q)|^2 \\ &= \int \frac{dt}{t} \int dz_a \int ds_{ai} \int \frac{dx_a}{x_a} \int \frac{d\phi_i}{2\pi} \frac{2p_{ai}p_j}{s_{aij}} \\ & \quad \times \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{P_{(aij)a}(p_a, p_i, p_j)}{s_{aij}} \int d\Phi(\tilde{p}_{aij}, \tilde{p}_k | q) |M_n(\tilde{a}i\tilde{j}, \tilde{k} | q)|^2 \end{aligned}$$

- ▶ Write as differential branching probability

$$\frac{d \ln \Delta_{(aij)a}^{1 \rightarrow 3}}{d \ln t} = \int dz_a z_a \int ds_{ai} \int \frac{dx_a}{x_a} \int \frac{d\phi_i}{2\pi} \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{P_{(aij)a}(p_a, p_i, p_j)}{s_{aij}^2 / 2 p_{ai} p_j}$$

- ▶ LO PS accounts for iterated collinear limit, hence we must subtract

$$\frac{d \ln \Delta_{(aij)a}^{(1 \rightarrow 2)^2}}{d \ln t} = \int dz_a z_a \int \frac{ds_{ai}}{s_{ai}} \int \frac{d\xi}{\xi} \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{\sum_{(ai)} P_{(aij)(ai)}^{(0)}(\xi) P_{(ai)a}^{(0)}(z_a/\xi)}{s_{aij} / 2 p_{ai} p_j}$$

- ▶ Simplest possible configuration  $q \rightarrow q'$  [Catani,Grazzini] hep-ph/9908523

$$P_{qq'} = \frac{1}{2} C_F T_R \frac{s_{aij}}{s_{ai}} \left[ -\frac{t_{ai,j}^2}{s_{ai}s_{aij}} + \frac{4z_j + (z_a - z_i)^2}{z_a + z_i} + (1 - 2\varepsilon) \left( z_a + z_i - \frac{s_{ai}}{s_{aij}} \right) \right]$$

where  $(z_a + z_i) t_{ai,j} = 2(z_a s_{ij} - z_i s_{aj}) + (z_a - z_i) s_{ai}$

- ▶ Apparent collinear singularity in  $s_{ai}$  that cancels upon azimuthal averaging against iterated LO splitting
- ▶ But integrand locally divergent  $\rightarrow$  not amenable to MC simulation
- ▶ Solved by subtraction of spin-correlated LO splitting functions

[Somogyi,Trocsanyi,del Duca] hep-ph/0502226

$$P_{qg}^{\mu\nu} = C_F \left[ -2 \frac{z}{1-z} \frac{k_T^\mu k_T^\nu}{k_T^2} + \frac{1-z}{2} \left( -g^{\mu\nu} + \frac{p^\mu n^\nu + p^\nu n^\mu}{np} \right) \right]$$

$$P_{gq}^{\mu\nu} = T_R \left[ -g^{\mu\nu} + 4z(1-z) \frac{k_T^\mu k_T^\nu}{k_T^2} \right]$$

- ▶ Leads to additional subtraction term

$$\Delta P_{qq'} = C_F T_R \frac{4z_a z_i z_j}{(1-z_j)^3} (1 - 2 \cos^2 \phi) , \quad \cos \phi = \frac{s_{ai}s_{jk} + s_{ak}s_{ij} - s_{aj}s_{ik}}{2\sqrt{s_{ai}s_{ak}s_{ij}s_{jk}}}$$

- ▶ Reference for  $q \rightarrow q'$  upon integration over  $s_{ai}, x_a, \phi_j$  given by NLO kernel

$$P_{qq'}(z) = C_F T_R \left[ (1+z) \log^2 z - \left( \frac{8}{3} z^2 + 9z + 5 \right) \log z + \frac{56}{9} z^2 + 4z - 8 - \frac{20}{9z} \right]$$

- ▶ So far we have

$$P_{qq'}(z) = -C_F T_R \left[ 5(1-z) + 2(1+z) \log z \right]$$

- ▶ The difference lies in  $\mathcal{O}(\varepsilon^0)$  contributions from renormalization

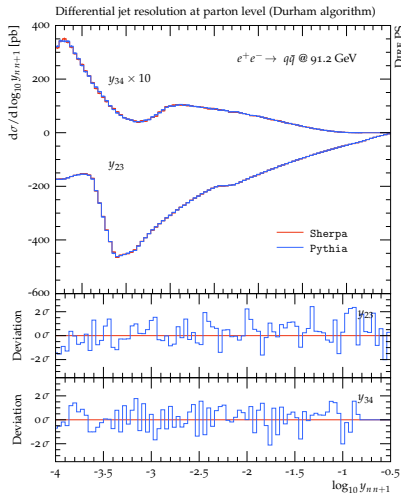
$$\Delta P_{qq'}^{(R)} = \int_{z_a} \frac{d\xi}{\xi} C_F \left( \frac{1+(1-\xi)^2}{\xi} \log(\xi(1-\xi)) + \xi \right) P_{gq}(z_a/\xi)$$

- ▶ ... and in integrating the iterated LO kernels over the  $D$ -dimensional  $1 \rightarrow 3$  phase space as required for local subtraction vs. integrating over the  $D$ -dimensional  $1 \rightarrow 2$  phase space as required for  $\Delta P_{qq'}^{(R)}$

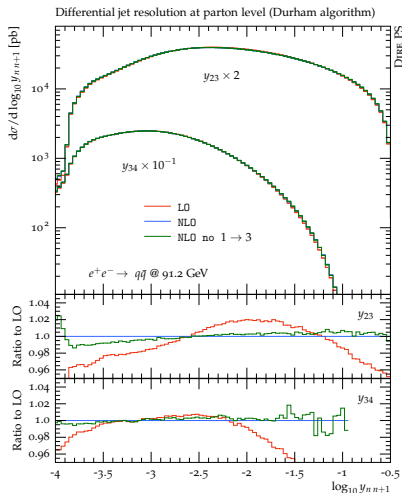
$$C_F T_R \left[ \frac{1+z_j^2}{1-z_j} + \left( 1 - 2 \frac{z_a z_i}{(z_a + z_i)^2} \right) \left( 1 - z_j + \frac{1+z_j^2}{1-z_j} \right) \left( \log(z_a z_i z_j) - 1 \right) \right]$$

- ▶ Contributions due to  $D$ -dimensionality of phase space must not spoil differential radiation pattern (we live in 4D)
- ▶ Hence simulate as endpoint contributions:
  - ▶ Generated using triple collinear phase space, but retroactively projected onto  $s_{ai} = 0$
  - ▶ Guarantees phase-space coverage identical to fully differential simulation
  - ▶ Remainder taken care of by PS unitarity
- ▶ Despite subtraction terms not being accurate for current evolution variable, this is a general scheme for including NLO kernels in parton showers
- ▶ Most importantly, we never have to compute integrals analytically
- ▶ Natural extension to kernels with virtual corrections  
→ “MC@NLO” inside the shower





► 1  $\rightarrow$  3 emission test (one only)



► NLO vs LO comparison

## So far

- ▶ Developed MC algorithm to implement  $2 \rightarrow 4$  splittings that recovers integrated NLO splitting functions for  $q \rightarrow q' / q \rightarrow \bar{q}$
- ▶ Added integrated splitting functions in all other cases
- ▶ Cross-validated all implementations in FF/FI branchings

## To-do list

- ▶ Triple collinear splitting function implementation for IF/II branchings
- ▶ Addition of corrections to account for evolution in dipole- $k_T$
- ▶ Implementation of  $2 \rightarrow 4$  splittings for all NLO kernels

- ▶ Problem in NLO splitting kernels, sub-leading color terms, etc. lies in negative weights  $\rightarrow$  no-emission probability *locally* exceeds unity
- ▶ Recall standard veto algorithm:  $\mathcal{P}_{\text{no}}(t, t') = \exp\{F(t) - F(t')\}$   
Exact MC solution  $t = F^{-1}[F(t') + \log R]$ ,  $R$  – random number
- ▶ Don't want or can't compute  $F(t) = -\int_t d\bar{t} f(\bar{t})$ ,  
instead find simple function  $g(t) > f(t)$  with integral  $G(t)$
- ▶ Generate points according to  $g(t)$  and accept with  $f(t)/g(t)$

Standard probability for **one acceptance** with  $n$  **rejections**

$$\frac{f(t)}{g(t)} g(t) \exp\left\{-\int_t^{t_1} d\bar{t} g(\bar{t})\right\} \prod_{i=1}^n \left[ \int_{t_{i-1}}^{t'_i} dt_i \left(1 - \frac{f(t_i)}{g(t_i)}\right) g(t_i) \exp\left\{-\int_{t_i}^{t_{i+1}} d\bar{t} g(\bar{t})\right\} \right]$$

Split weight into MC and **analytic** part using auxiliary function  $h(t)$

$$\frac{f(t)}{h(t)} g(t) \exp\left\{-\int_t^{t_1} d\bar{t} g(\bar{t})\right\} \prod_{i=1}^n \left[ \int_{t_{i-1}}^{t'_i} dt_i \left(1 - \frac{f(t_i)}{h(t_i)}\right) g(t_i) \exp\left\{-\int_{t_i}^{t_{i+1}} d\bar{t} g(\bar{t})\right\} \right]$$

$$w(t, t_1, \dots, t_n) = \frac{h(t)}{g(t)} \prod_{i=1}^n \frac{h(t_i) g(t_i) - f(t_i)}{g(t_i) h(t_i) - f(t_i)}$$

Looks trivial, surprisingly it's not: It allows to

- ▶ Resum sub-leading color terms in MC@NLO and POWHEG  
[Krauss,Schönherr,Siegert,SH] arXiv:1111.1220
- ▶ Implement higher-order splitting functions in parton showers  
[Krauss,Prestel,SH] any time soon
- ▶ Use PDFs with negative values in parton showers  
[Prestel,SH] arXiv:1506.05057
- ▶ Enhance branching probabilities in parton showers  
[Schumann,Siegert,SH] arXiv:0912.3501, [Lönnblad] arXiv:1211.7204
- ▶ Reweight parton showers [Bellm,Plätzer,Richardson,Siódmok,Webster] arXiv:1605.08256  
[Mrenna,Skands] arXiv:1605.08352, [Bothmann,Schönherr,Schumann] arXiv:1606.08753

[SH] TBP?

- ▶ Can view new shower model as modification of CS subtraction
- ▶ IR counterterms computed and implemented in Sherpa (improved cancellation in  $pp \rightarrow h + j$  due to regulated  $1/z$  terms)

- ▶ Sherpa MC@NLO based on exponentiation of CS dipole subtraction terms

[Krauss,Siegert,Schönherr,SH]

arXiv:1111.1220, arXiv:1208.2815

- ▶ Dire modified CS subtraction automatically available for MC@NLO matching
- ▶ Interesting differences due to evolution variables and kernels

