Precision Event Generation for the LHC

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UCB Seminar

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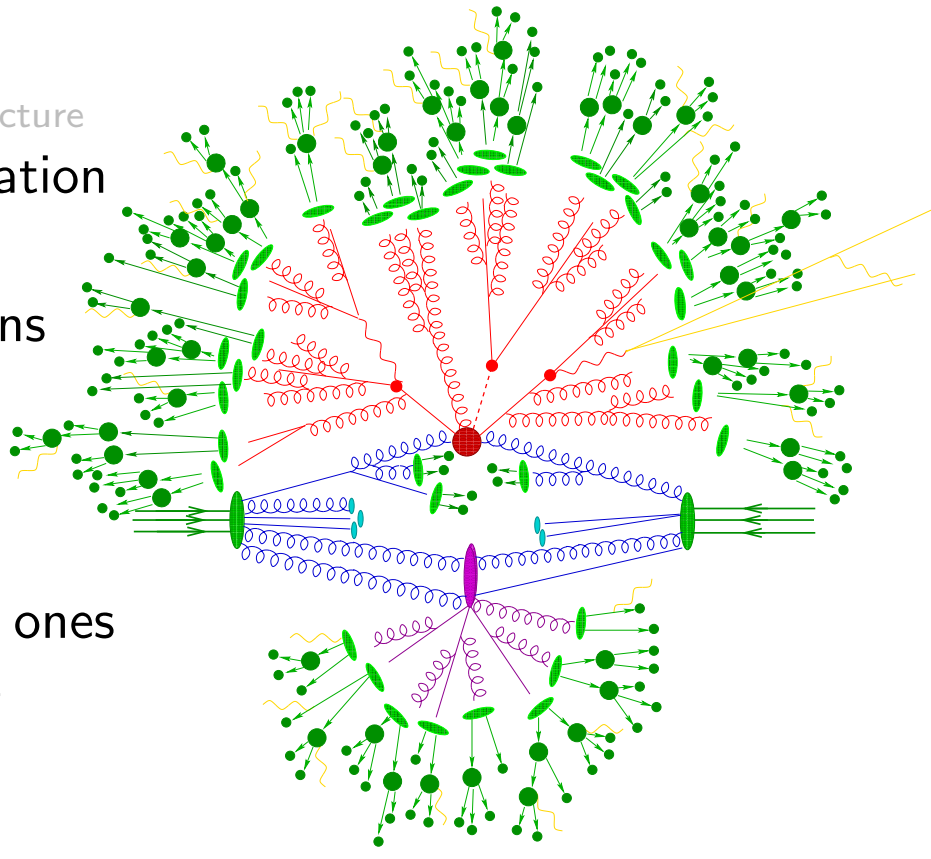
Event generation in standard Monte Carlo

1. Matrix Element (ME) generators red blobs
   simulate "central" part of the event
2. Parton Showers (PS) red & blue tree structure
   produce additional "hard" QCD radiation
3. Multiple interaction models purple blob
   simulate "secondary hard" interactions
4. Fragmentation models light green blobs
   hadronize QCD partons
5. Hadron decay modules dark green blobs
   decay primary hadrons into observed ones
6. Photon emission generators yellow stuff
   simulate additional QED radiation



Will focus on general-purpose MC plus NLO ME generators in this talk

Disclaimer: No attempt to present full status of the field

Find more in recently published review [Buckley et al.] Phys.Rept.504(2011)145

The hard matrix element

Event generation starts with the computation of $\hat{\sigma}$

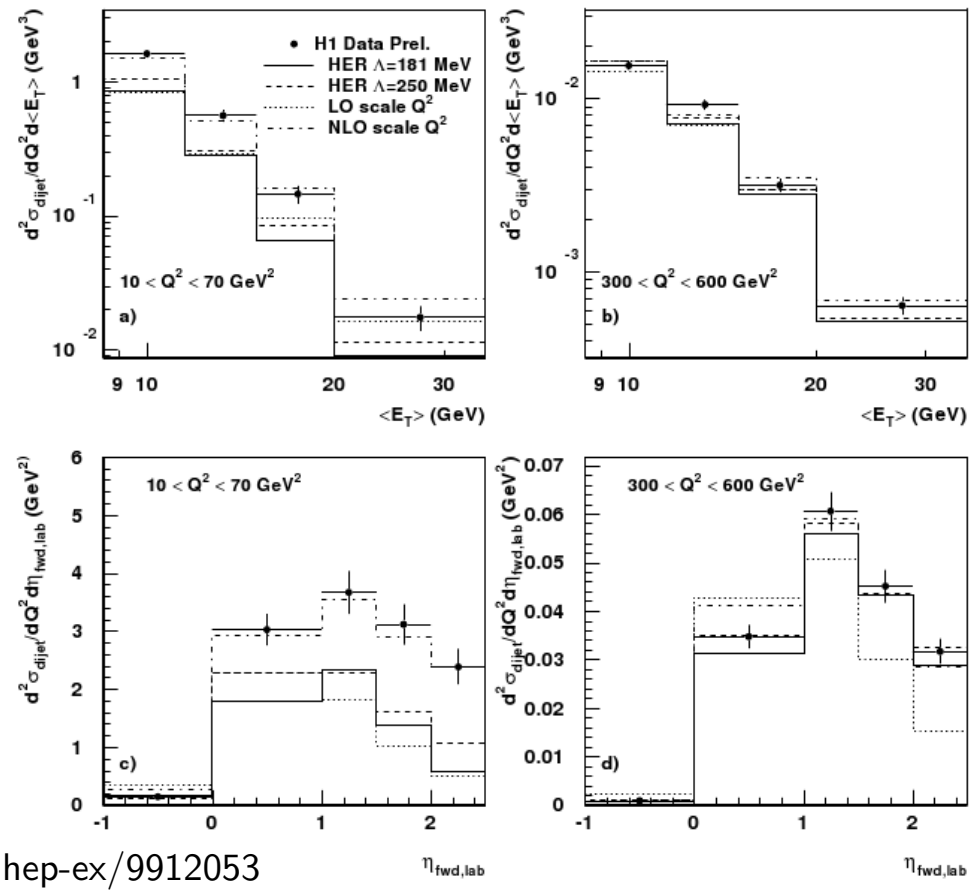
$$\hat{\sigma}_a = \int \mathrm{d}\Phi_n \frac{\mathrm{d}\hat{\sigma}_a}{\mathrm{d}\Phi_n}$$

What's the tricky point of this simple-looking formula ?

- Differential cross section $\mathrm{d}\hat{\sigma}_a/\mathrm{d}\Phi_n$
  difficult to obtain for large number of legs n
  solved problem at tree-level, newly emerging techniques at 1-loop
- $\mathrm{d}\Phi_n$ implies high-dimensional integral
  $\mathrm{d}\hat{\sigma}_a/\mathrm{d}\Phi_n$ most commonly sharply peaked
  much more problematic than ME generation because
  suitable phase space mappings must be found for arbitrary ME
- "Internal" degrees of freedom helicity & color
  num. effort for sum is (naively) $\sim 2^n$ / $3^n - 8^n$
  sampling can be much more efficient, depending on n

Simple solution: Employ leading-order ME and produce more partons with PS
$\rightarrow$ **Traditional "LO⊗PS" method, around for ∼20-30 years**

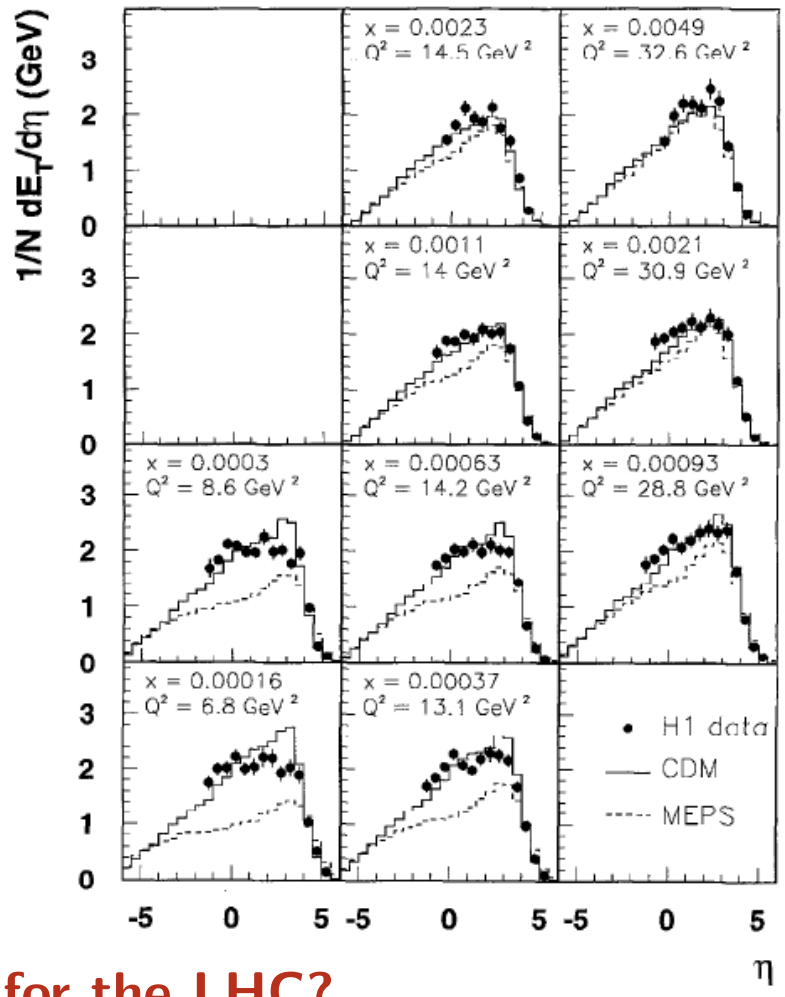
LO⊗PS predictions for DIS



hep-ex/9912053

Example: Energy flow PLB356(1995)118

- MEPS (DGLAP) fails for low Q^2
- CDM partially agrees with data



Example: Di-jet events EPJC19(2001)289

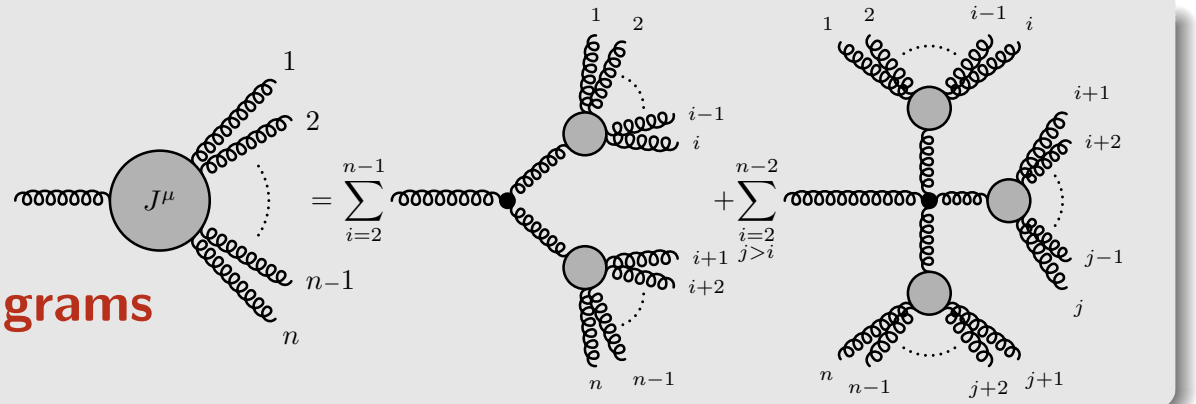
- PS fails to describe jet spectra
- Low-Q^2 region especially problematic

Should we really rely on such simulations for the LHC?

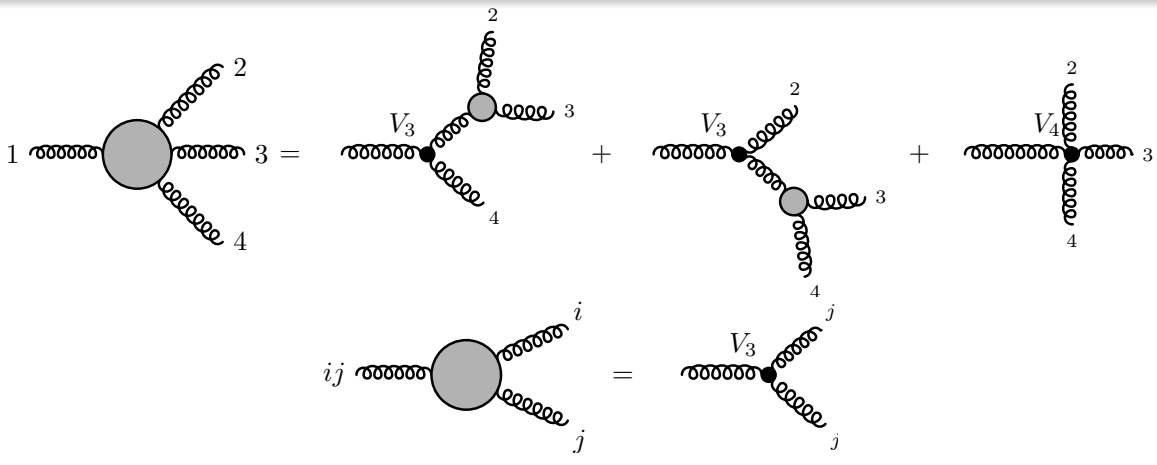
The quest for many jets

Example: ME generation in n-gluon process

- ME is sum of all possible Feynman graphs
- **Use recursive equation to find contributing diagrams**



Example: Diagrams for $g(1)g(2) \to g(3)g(4)$



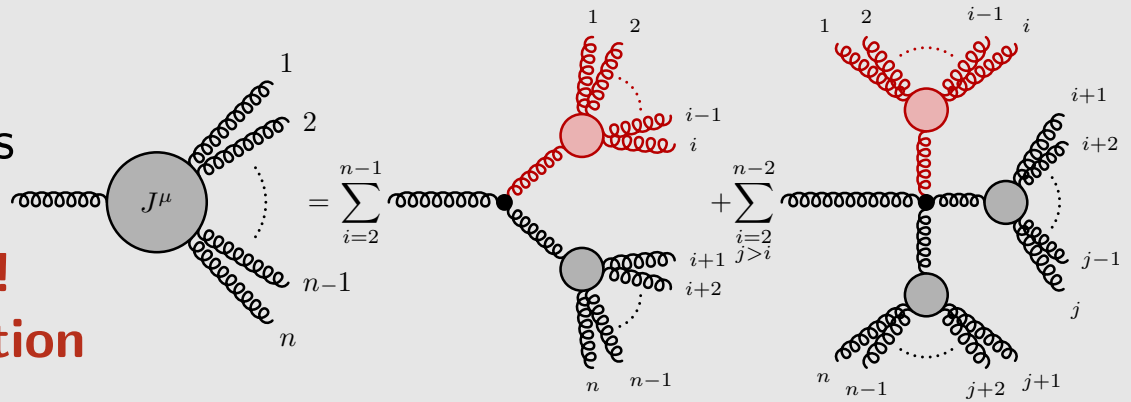
$\to$ Translate diagrams into "helicity amplitudes" [Ballestrero,Maina] PLB350(1995)225
using the fact that L/R spinors are basic representations of the Lorentz group
$\to$ Any scattering amplitude can be expressed in terms of spinor products ("almost" Lorentz invariants)

Note: No more than strings of up to four spinor products needed
alleviates computation due to reduced number of building blocks [Dixon] hep-ph/9601359

The quest for many jets

Example: ME generation in n-gluon process [Berends,Giele] NPB306(1988)759

- ME built from off-shell currents joined by vertices
  off-shell current $\leftrightarrow$ internal
- **Read from right to left!**
  no redundant computation



Example: Computing $g(1)g(2) \to g(3)g(4)$

Step 1	$J_1 = \varepsilon(1)$	$J_2 = \varepsilon(2)$	$J_3 = \varepsilon(3)$	$J_4 = \varepsilon(4)$
Step 2	J_{12}	J_{13}	J_{23}	
Step 3	J_{123}			
Step 4	$A(1,2,3,4) = J_4^* \, J_{123}$			

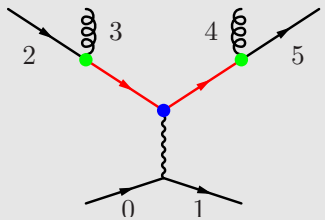
Performance of color dressed Berends-Giele recursion [Gleisberg,SH] JHEP08(2006)062

gg $\to$ ng	Cross section [pb]				
n	8	9	10	11	12
$\sqrt{s}$ [GeV]	1500	2000	2500	3500	5000
Comix	0.755(3)	0.305(2)	0.101(7)	0.057(5)	0.026(1)
PRD67(2003)014026	0.70(4)	0.30(2)	0.097(6)		
NPB539(1999)215	0.719(19)				

The quest for many jets

Commonly used method to evaluate multi-particle phase space

- Guess peak structure of integrand from dynamics [Byckling,Kajantie] NPB9(1969)568



$$\leftrightarrow \quad D_{iso}(23,45) \otimes P_0(23) \otimes P_0(45) \otimes D_{iso}(2,3) \otimes D_{iso}(4,5)$$

- Combine channels corresponding to single diagrams into multi-channel and optimize [Kleiss,Pittau] CPC83(1994)141
- Refine single integration channels with VEGAS [Lepage] JCP27(1978)192

Only propagators and vertices introduce integration channels !

→ phase space generation technique corresponds to ME generation diagrammatic and recursive way!

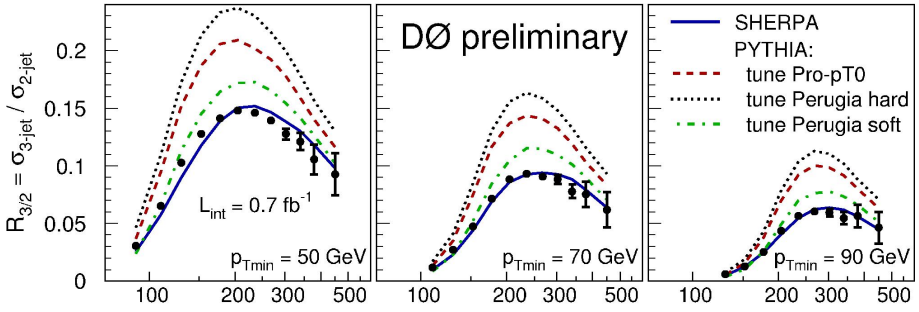
Other, less optimized / less general techniques exist, like Rambo & HAAG

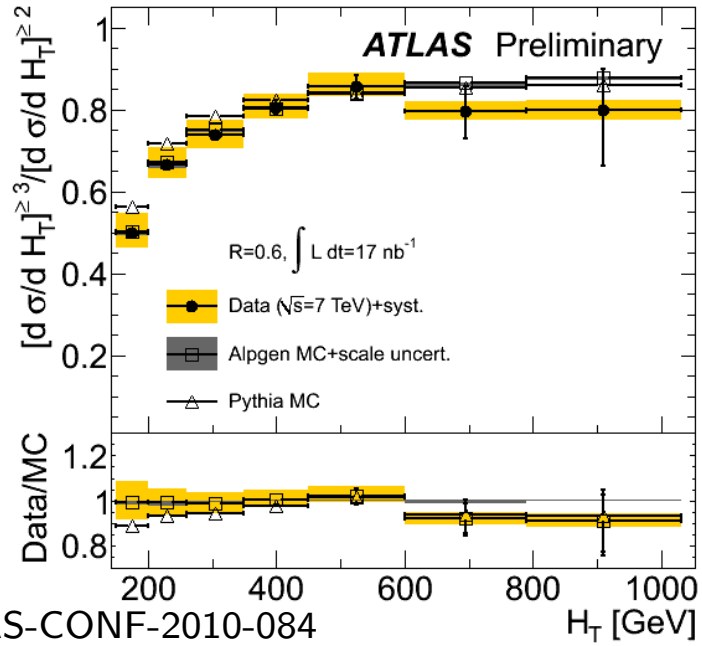
Correlations and interferences within ME often render optimization cumbersome
Color- and/or helicity sampling impairs convergence

Most general-purpose phase-space generators employ above technique
AMEGIC++, Comix, MADEVENT, PHEGAS, WHIZARD, ...

Virtues of matrix element generators

Ratio of 3- & 2-jet rate DØ Note 6032-CONF



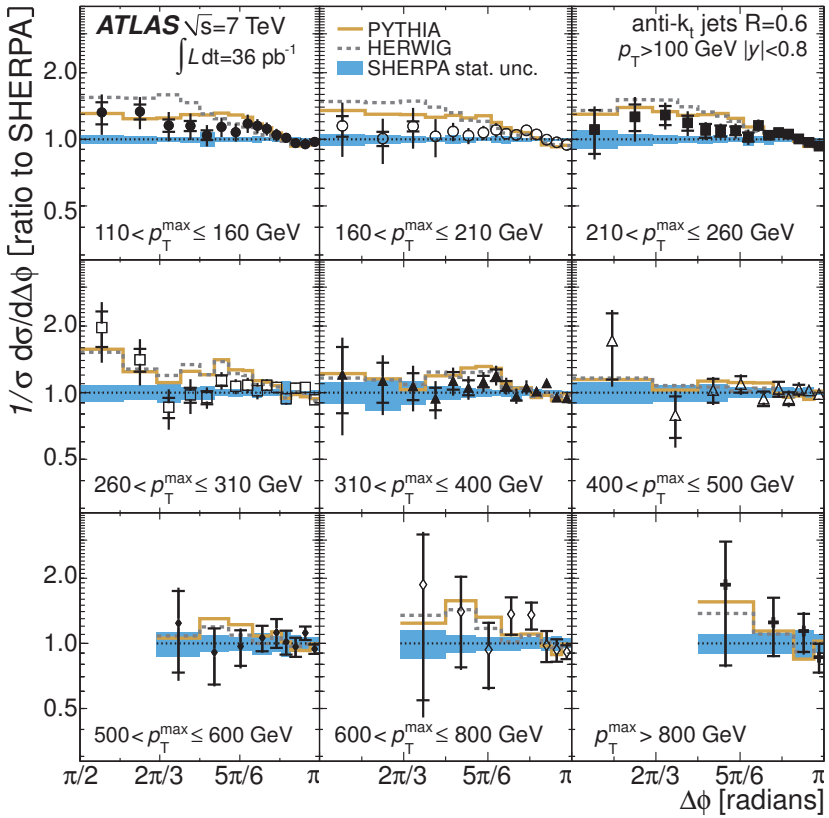


ATLAS-CONF-2010-084

Multi-leg ME⊗PS "standard" by now

Higher-order ME improve predictions for jet correlations & relative rates

Dijet decorrelation (MC vs. data)



[ATLAS] arXiv:1102.2696

On to higher precision

Improving predictions for hard scattering processes
requires efficient computation of QCD one-loop corrections

Obvious idea: Share the workload between tree-level and loop-level programs

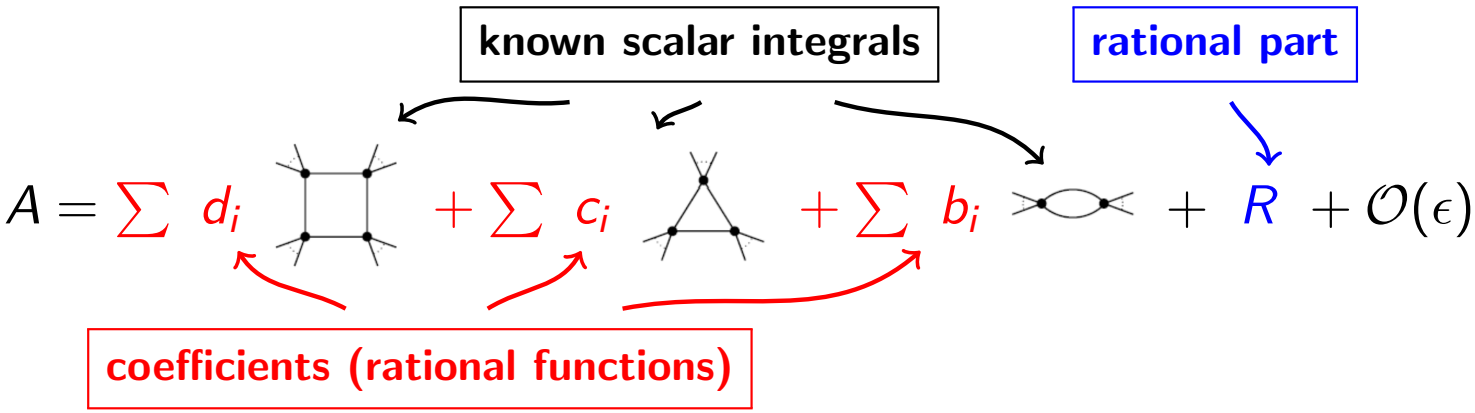
$$\sigma_{\mathrm{NLO}} = \int \mathrm{d}\Phi_{\mathrm{B}} \sum \left\{ \mathrm{B} + \tilde{\mathrm{V}} + \mathrm{I} \right\} + \int \mathrm{d}\Phi_{R} \sum \left\{ \mathrm{R} - \mathrm{S} \right\}$$

Standardized interface exists as Les Houches accord [Binoth et al.] CPC181(2010)1612

- One-Loop Engines (OLEs) like BlackHat [Berger et al.] PRD78(2008)036003 or GOLEM [Binoth et al.] CPC180(2009)2317 provide virtual piece or more
- ME generator takes care of Born, real emission, subtraction phase-space integration and event generation

The quest for many jets reloaded

Use generalized unitarity to compute virtual corrections
Basic ideas: [Bern,Dixon,Dunbar,Kosower] NPB435(1995)59 NPB513(1998)3



Cut-constructible part of virtual amplitude reduced to scalar integrals
at integrand level $\rightarrow$ determine coefficients d, c & b from tree amplitudes
[Ossola,Papadopoulos,Pittau] NPB763(2007)147, [Forde] PRD75(2007)125019

Fully automated by BlackHat collaboration PRD78(2008)036003
[Berger,Bern,Dixon,Febres-Cordero,Gleisberg,Forde,Ita,Kosower,Maître], also [Diana,Ozeren,SH]

Fast trees (from N=4 SYM, BG or BCFW) recycled into fast loops
Rational piece from D-dimensional unitarity or loop-level recursion

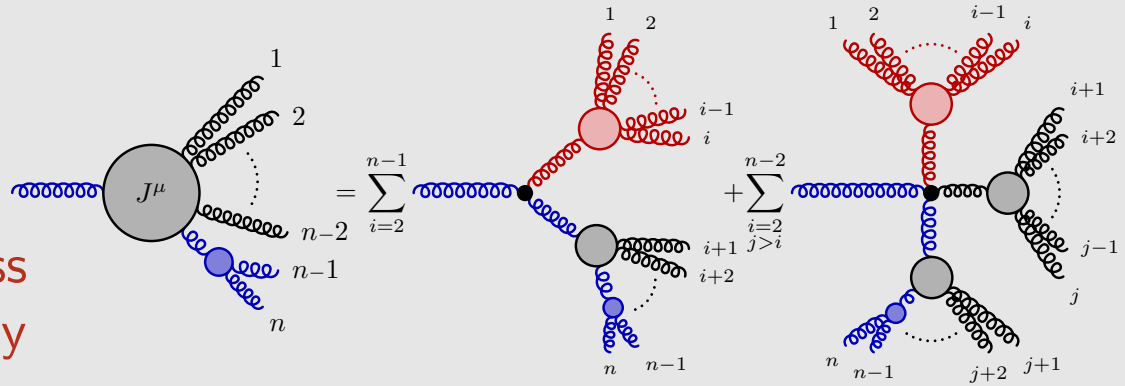
The quest for many jets reloaded

OLEs attacks higher and higher jet multiplicity
$\rightarrow$ need to compute real-radiation & infrared subtraction terms efficiently

Is Catani-Seymour dipole subtraction an obstacle for high multiplity?

Tree-level problem $\Rightarrow$ Extend CDBG recursion from [Duhr,Maltoni,SH] JHEP 08(2006)062

- Fix spectator parton as "final" leg in amplitude
- Recycle subamplitudes from real-radiation process and dipoles simultaneously



Use framework of existing ME generator Comix [Gleisberg,SH] JHEP 12(2008)039

- Real subtraction terms ✓
- Integrated subtraction terms

So far no indication that CS method is the main bottleneck!
Number of real subtraction terms grows approximately like N^3 i.e. very mild

The quest for many jets reloaded

$e^+e^- \to$jets (91.2 GeV) real-radiation & subtraction only

σ_{R-S} [pb]	Number of jets				
n k_T-jets	2	3 $\alpha_c = 0.1$	4 $\alpha_c = 0.03$	5 $\alpha_c = 0.01$	6 $\alpha_c = 0.003$
AMEGIC++/BlackHat	-357(2)	1667(17)	1047(8)	167(11)	–
Comix	-356(2)	1631(16)	**1056(11)**	**169(2)**	**12.01(6)**
Speedup*	0.63	0.52	**1.54**	**2.0**	–

$pp \to e^+e^-$+jets (7 TeV) real-radiation & subtraction only

σ_{R-S} [pb]	Number of jets					
n k_T-jets	0	1 $\alpha_c = 0.1$	2 $\alpha_c = 0.03$	3 $\alpha_c = 0.01$	4 $\alpha_c = 0.003$	5 $\alpha_c = 0.001$
AMEGIC++/BlackHat	30.2(1)	47.2(5)	93.0(9)	55.8(3)	?	–
Comix	30.0(1)	46.1(5)	**92.6(6)**	**54.8(4)**	**23.2(2)**	
Speedup*	0.4	0.46	**1.92**	**1.76**	?	–

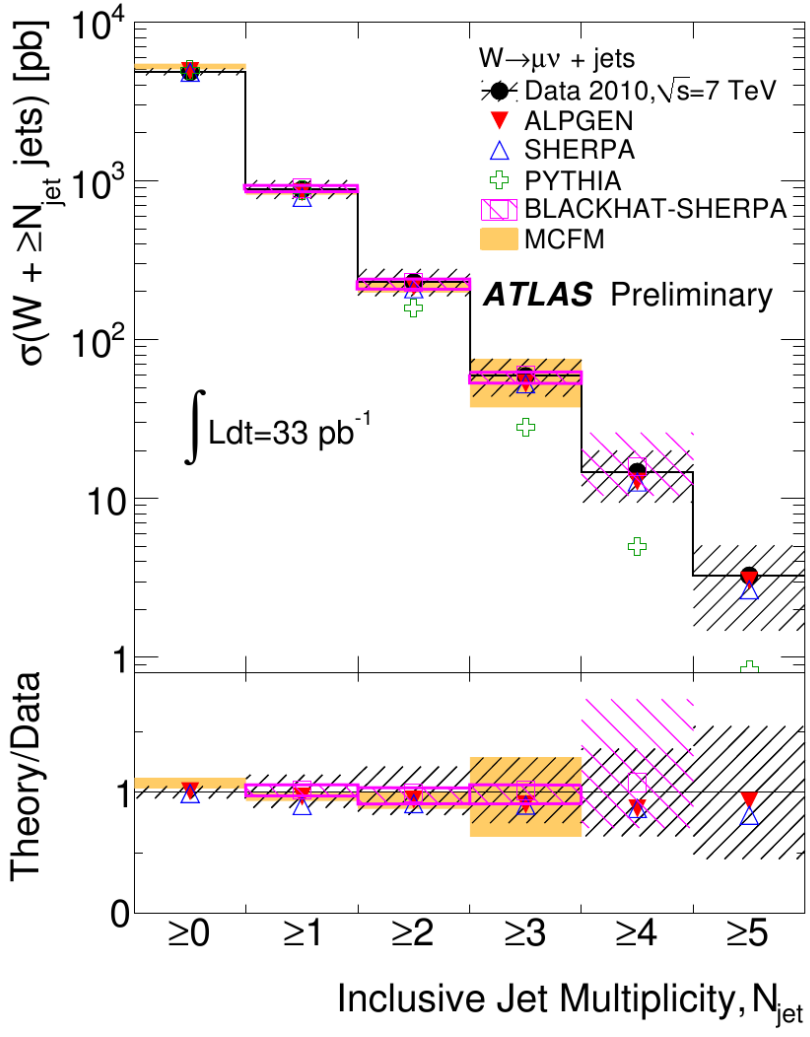
$pp \to e^+\nu_e$+jets (7 TeV) real-radiation & subtraction only

σ_{R-S} [pb]	Number of jets					
n k_T-jets	0	1 $\alpha_c = 0.1$	2 $\alpha_c = 0.03$	3 $\alpha_c = 0.01$	4 $\alpha_c = 0.003$	5 $\alpha_c = 0.001$
AMEGIC++/BlackHat	-200(1)	297(3)	576(6)	342(2)	?	–
Comix	-198(1)	297(3)	586(6)	**343(1)**	**143(1)**	**31.7(6)**
Speedup*	0.4	0.9	0.6	**3.39**	?	–

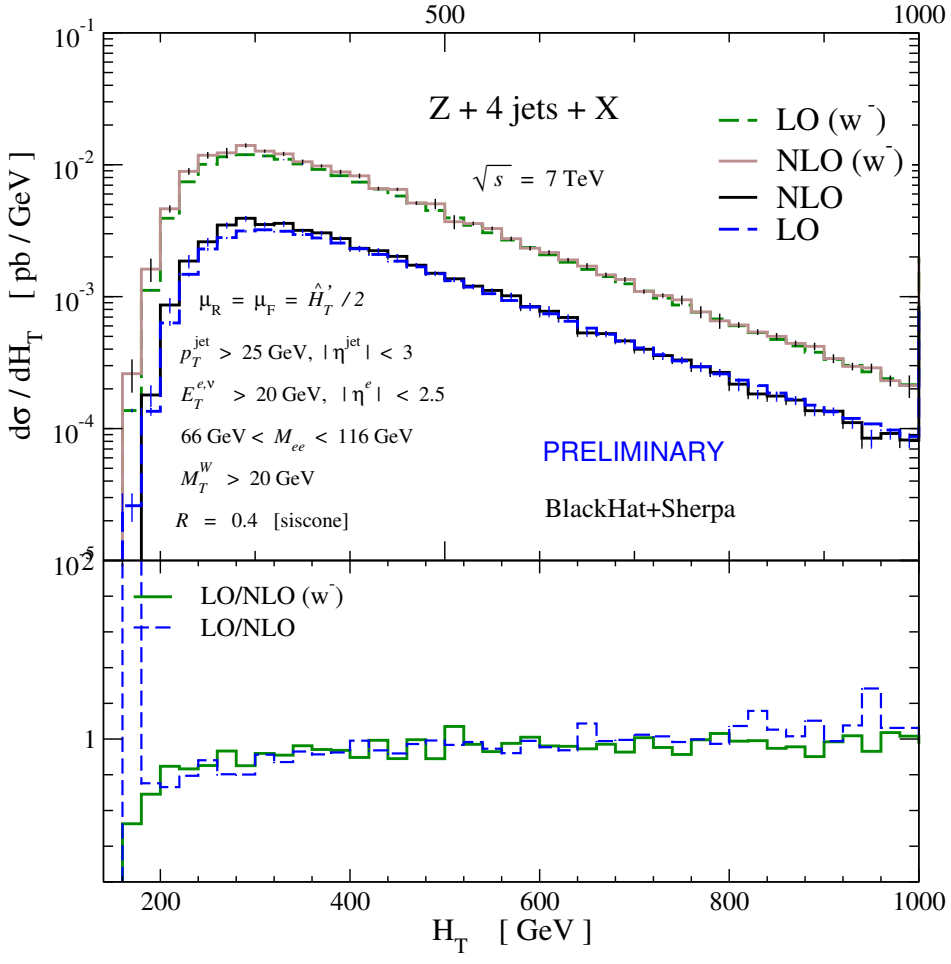
*Timing is for complete integration (i.e. matrix-element *and* phase-space)

The quest for many jets reloaded

Synergy between OLE and MC allows to attack e.g. W+3/4 jets, Z+3/4 jets



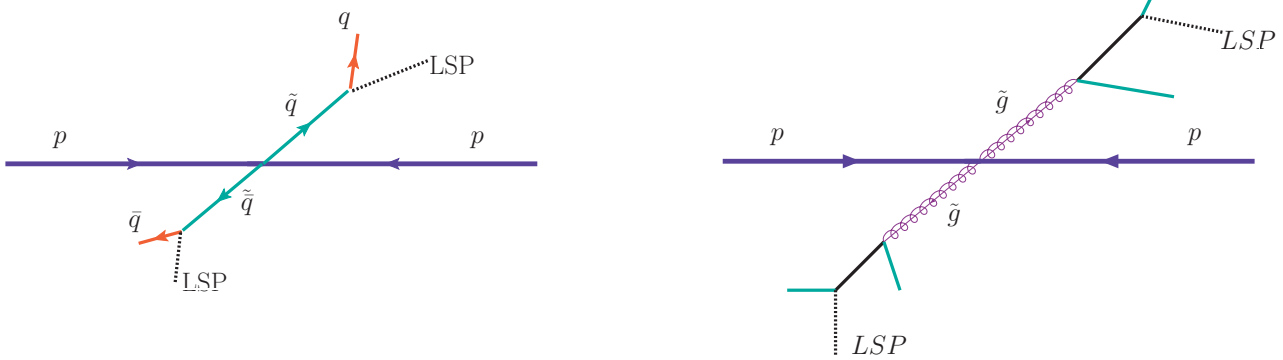
[Atlas] ATLAS-CONF-2011-060



[BlackHat] arXiv:1108.2229 [hep-ph]

NLO at work: Driving missing data

Generic SUSY signature: $\not{E}_T$+jets



$Z[\to \nu\bar{\nu}]$+jets irreducible background, need to calibrate

Use γ+jets (high statistics) and theory input [CMS] PAS SUS-08-002 PAS SUS-10-005

$$\sigma(Z \to \nu\bar{\nu}) = \sigma(\gamma) \times R_{Z/\gamma}$$

Must understand possible kinematical differences between Z+jets and γ+jets

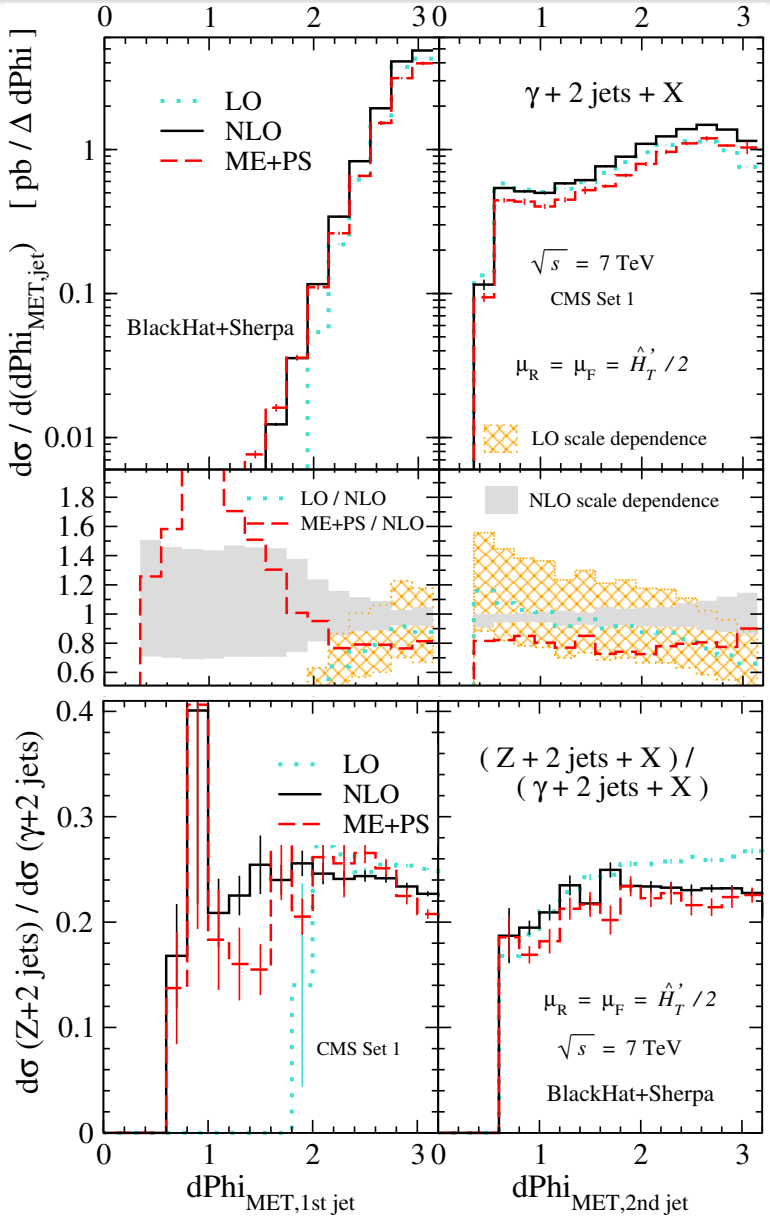
$\to$ **Use fully differential prediction for $R_{Z/\gamma}$ from BlackHat +SHERPA**

Theory uncertainty assessed by comparison with complementary approach

$\to$ Tree-level matrix-element parton-shower merging

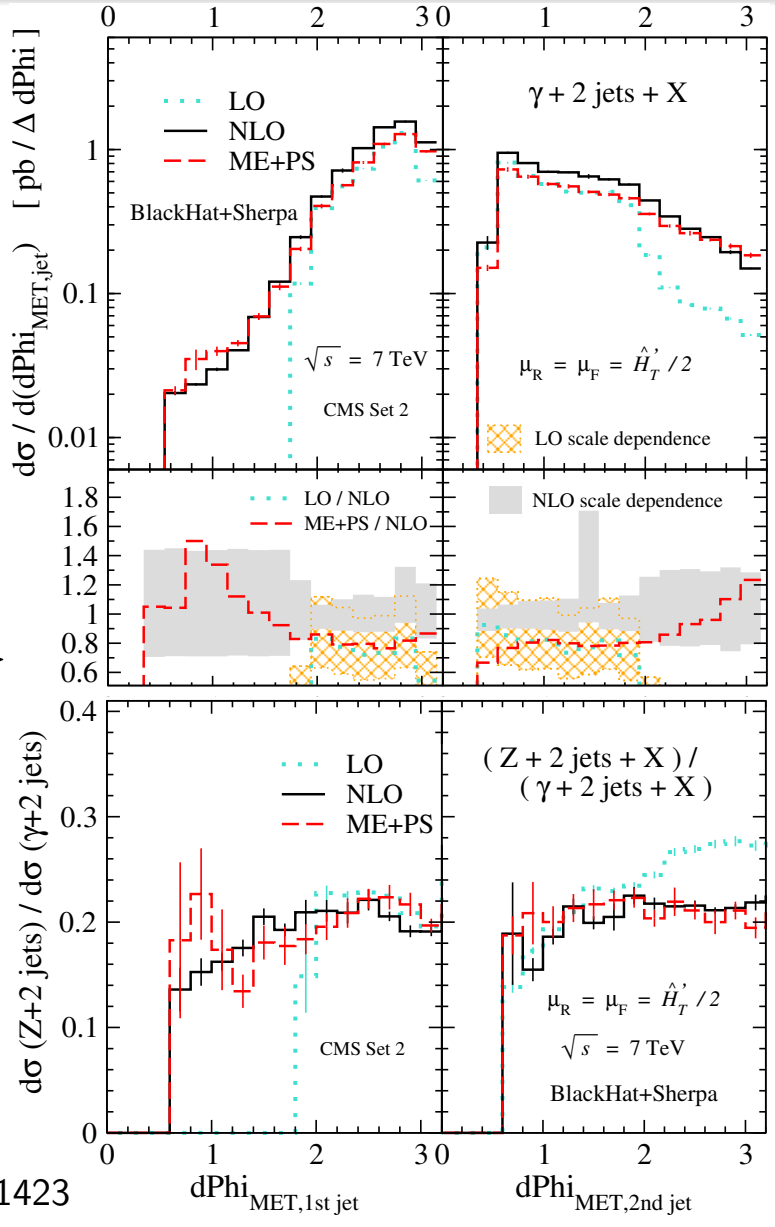
Started with Z/γ+2 jets to understand the problem, attacking Z/γ+3 jets next

NLO at work: Driving missing data



←Set 1 (high $\not{E}_T$) direct $\tilde{q}$ decays

Set 2 (high H_T) → cascades, soft LSP

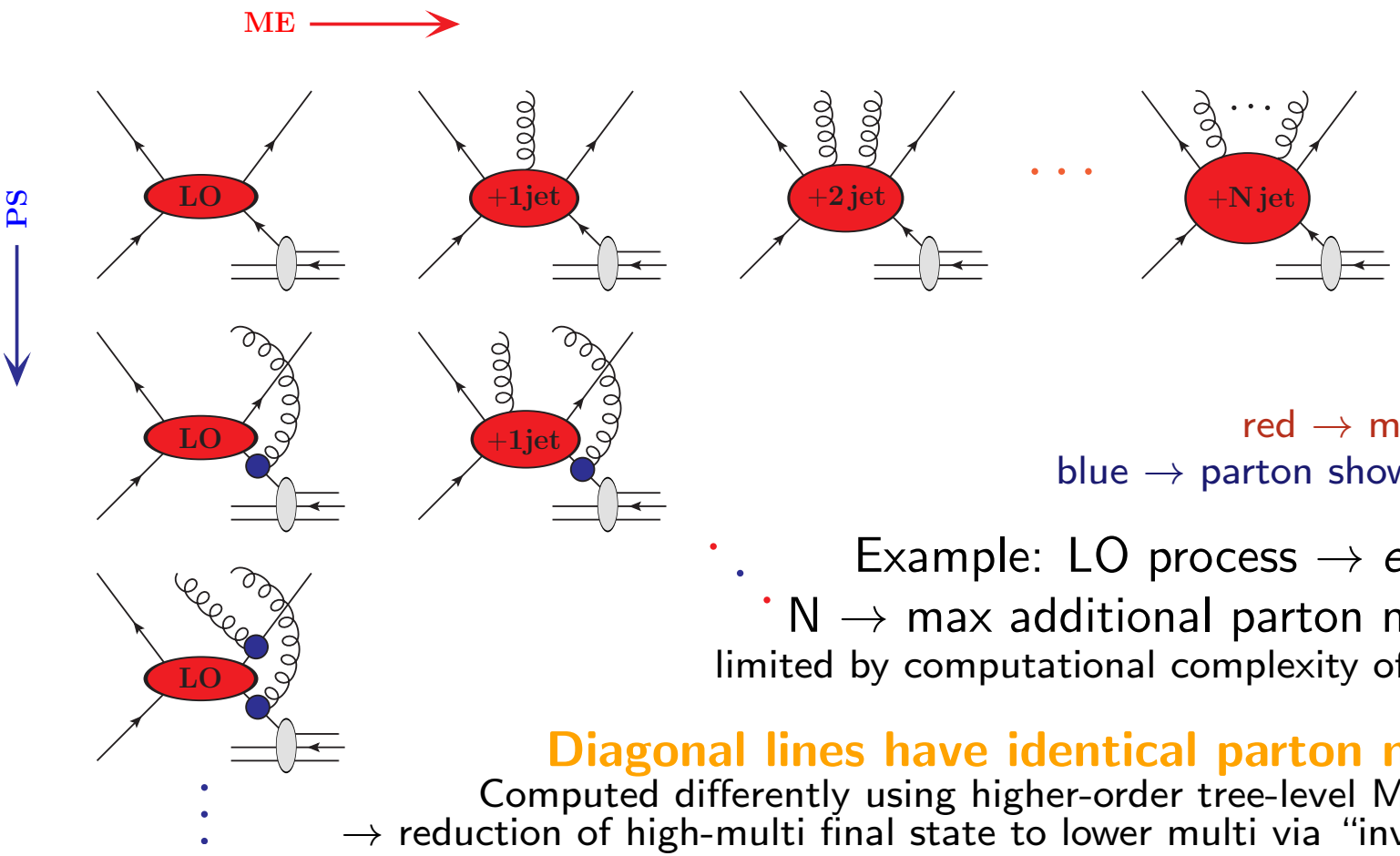


[BlackHat] arXiv:1106.1423

Results used by CMS to estimate $\lesssim 10\%$ BG uncertainty [CMS] arXiv:1106.4503

Connecting ME and PS - The tree-level case

Chance and challenge: ME and PS can simulate the same thing!



red → matrix element
blue → parton shower branching

Example: LO process → $e^{\pm}q \to e^{\pm}q$

N → max additional parton multi in ME
limited by computational complexity of the problem

Diagonal lines have identical parton multiplicity
Computed differently using higher-order tree-level ME or/and PS
→ reduction of high-multi final state to lower multi via "inverse" shower

ME⊗PS idea: Use ME/PS in regime of their respective strengths

ME → hard emissions / PS → soft/collinear regime JHEP11(2001)063, JHEP05(2009)053

Making ME⊗PS inclusive

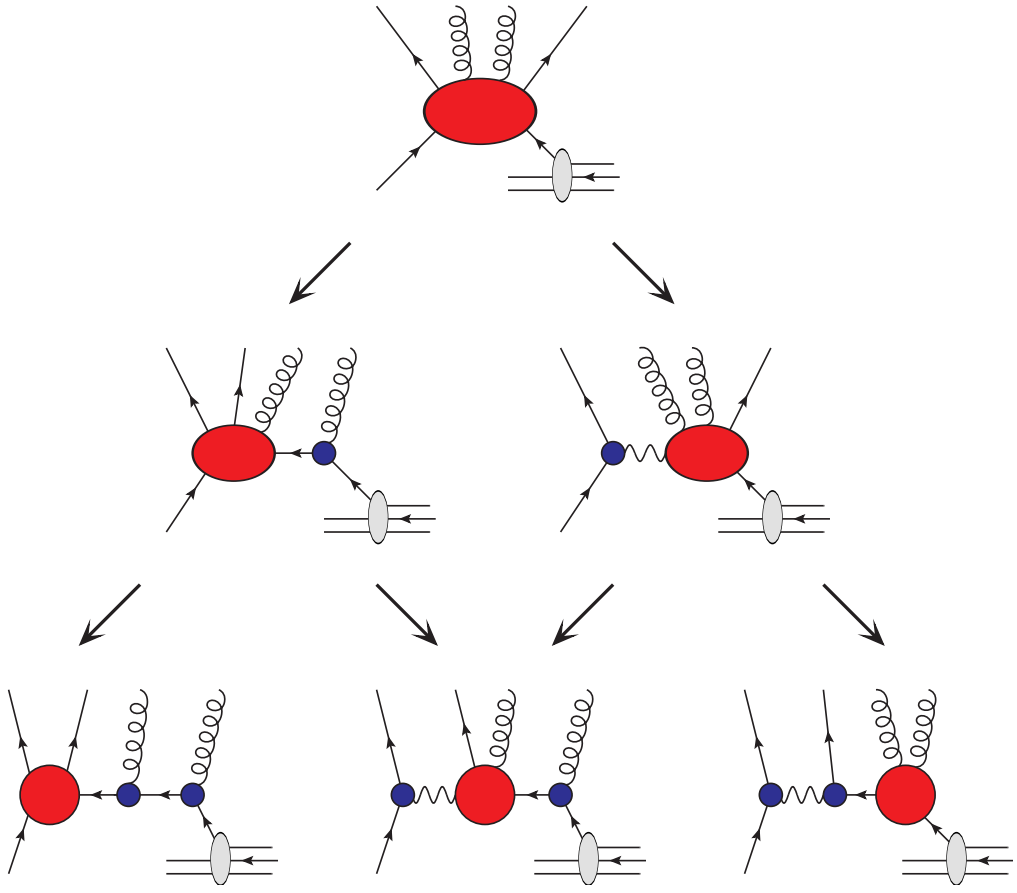
Matrix elements can have very different PS equivalents depending on kinematics

Must reduce full high-multi ME to either of these configurations in order to start PS

Radiation effects off intermediate legs must be modeled to account for Sudakov suppression → truncated PS

Method: PRD57(1998)5767, JHEP05(2009)053

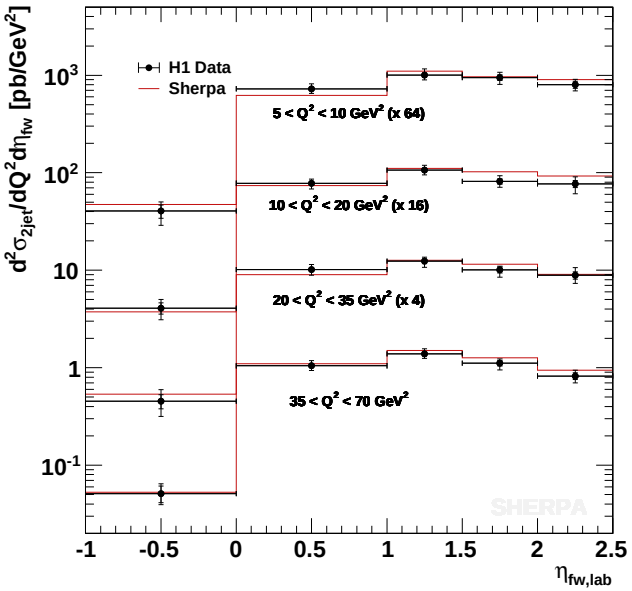
- Probability to identify splitting given by PS's branching eqns
- Reduced ME configuration defined by "inverted" PS splitting kinematics
- **Continue until** $2 \to 2$ **"core"**

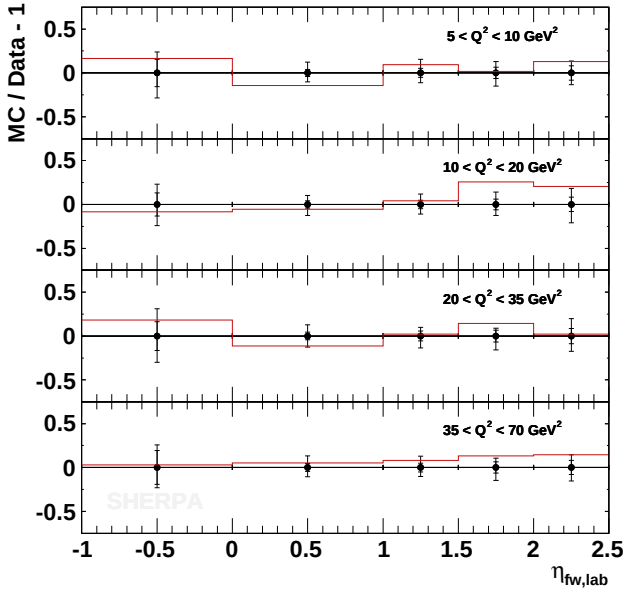


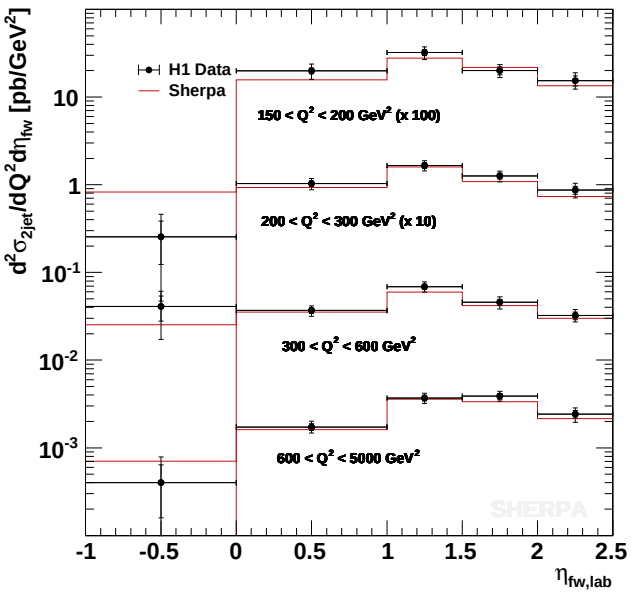
Core processes set the hardness scale of events $\to \mu_F$

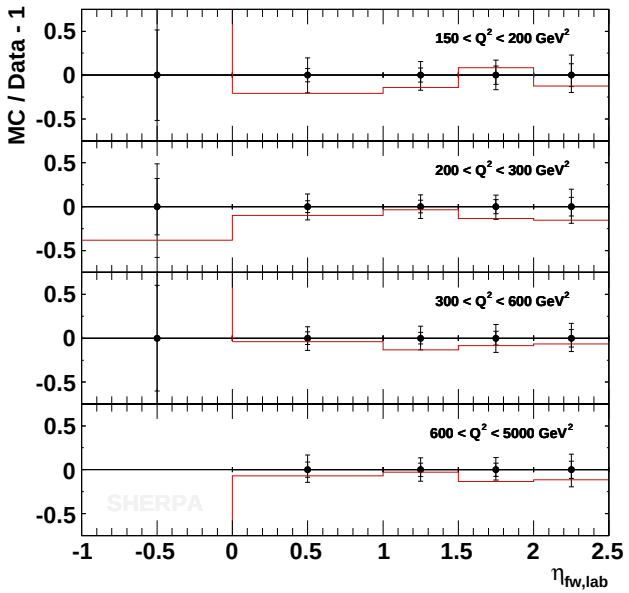
i.e. no scale should be larger than this PRD70(2004)114009, JHEP05(2009)053

Tree-level ME⊗PS at work: DIS



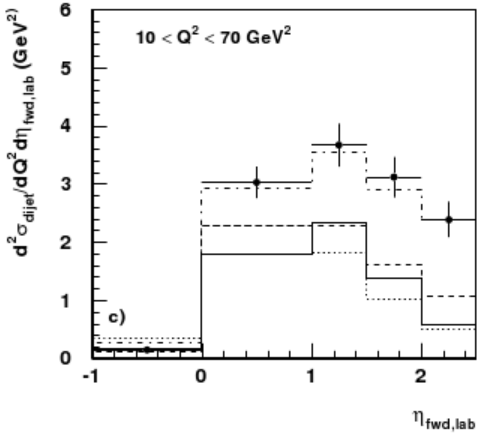


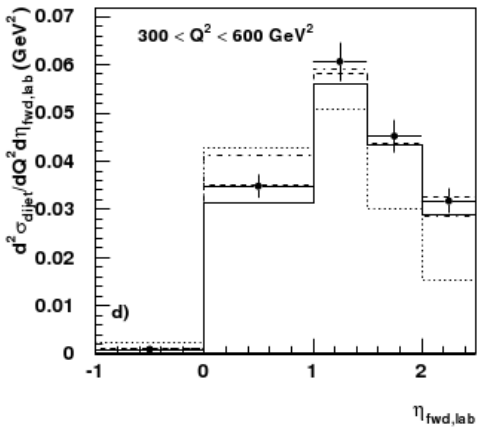




Compare to LO⊗PS results

→ MC status 1999 vs. 2010

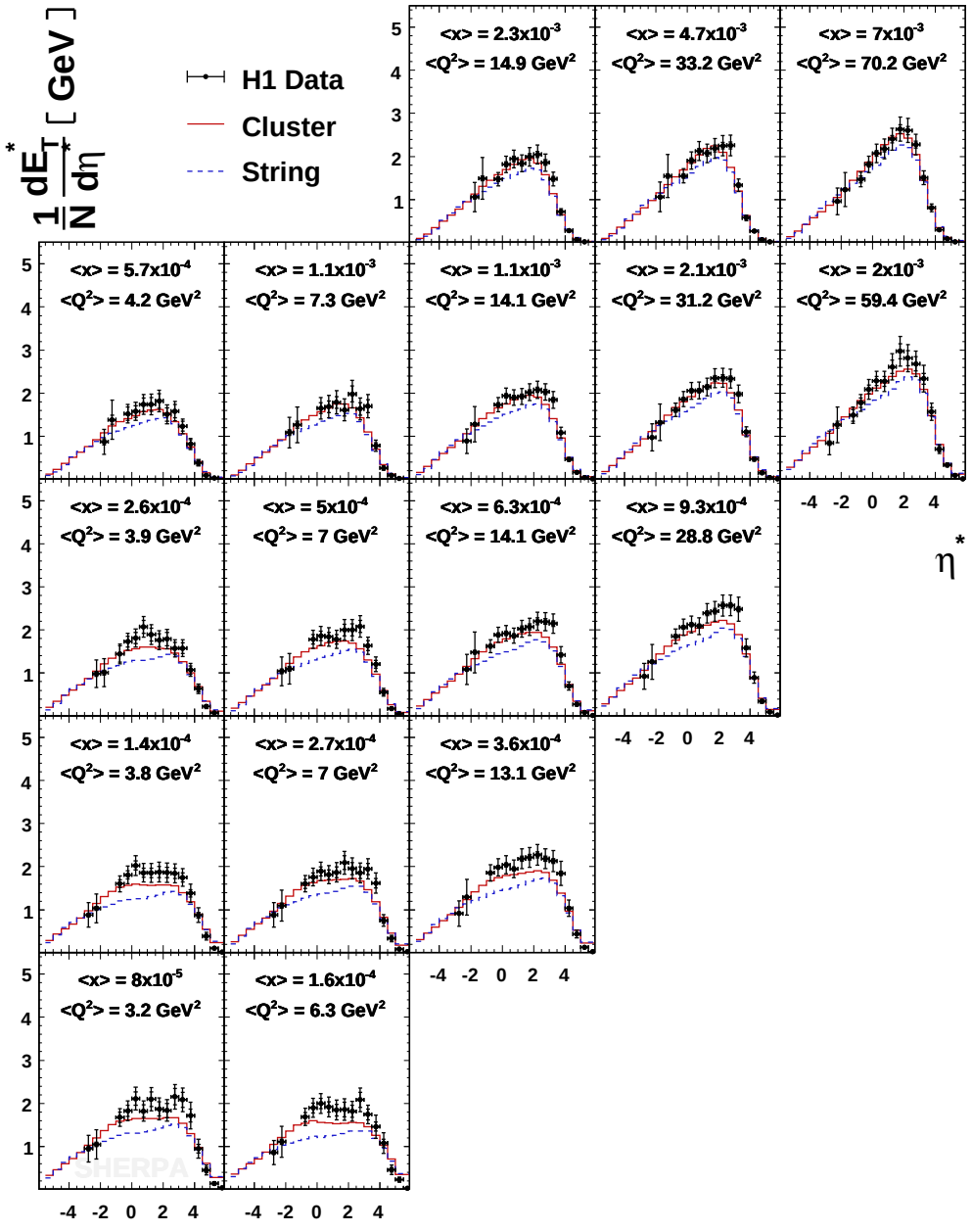




[Carli,Gehrmann,SH]

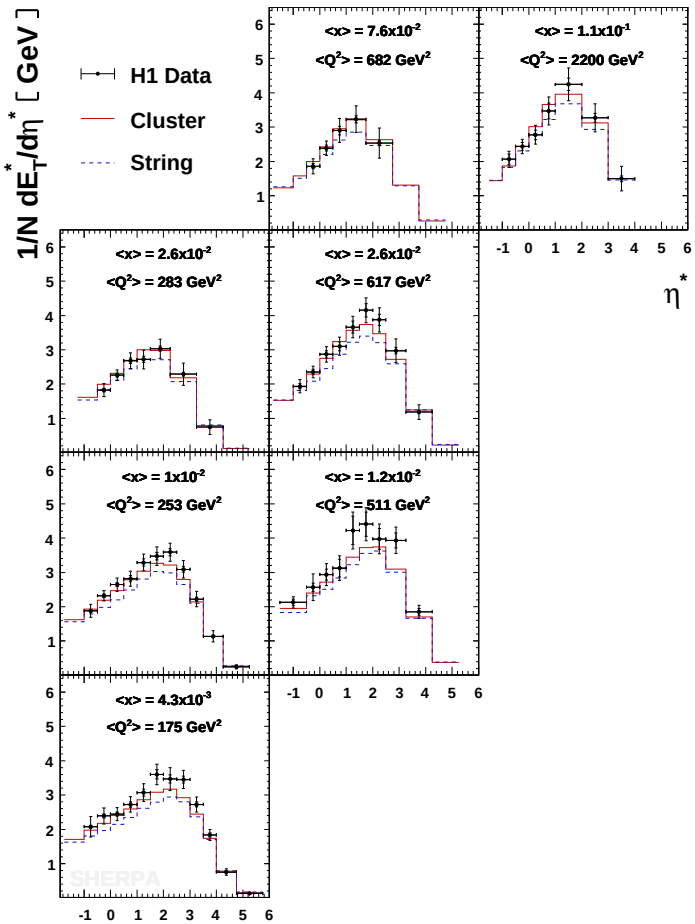
EPJC67(2010)73

Tree-level ME⊗PS at work: DIS



Transverse energy flow

Hadronization effects often large
but overall picture much improved

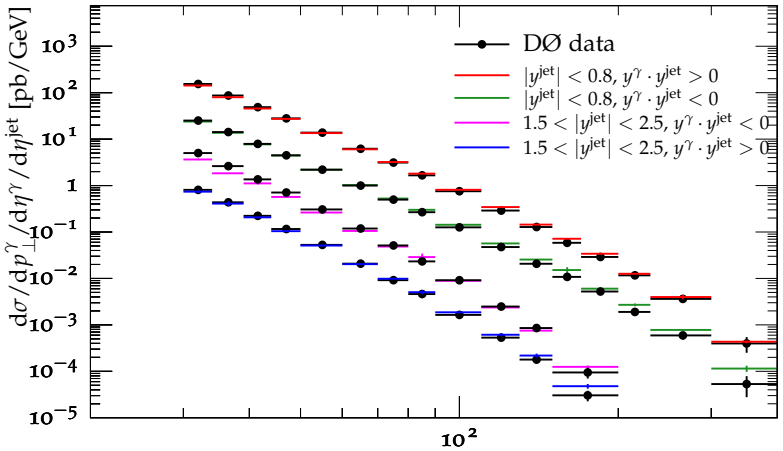


[Carli,Gehrmann,SH] EPJC67(2010)73

Tree-level ME⊗PS at work: Prompt photons

Photon p_T spectra PLB666(2008)435

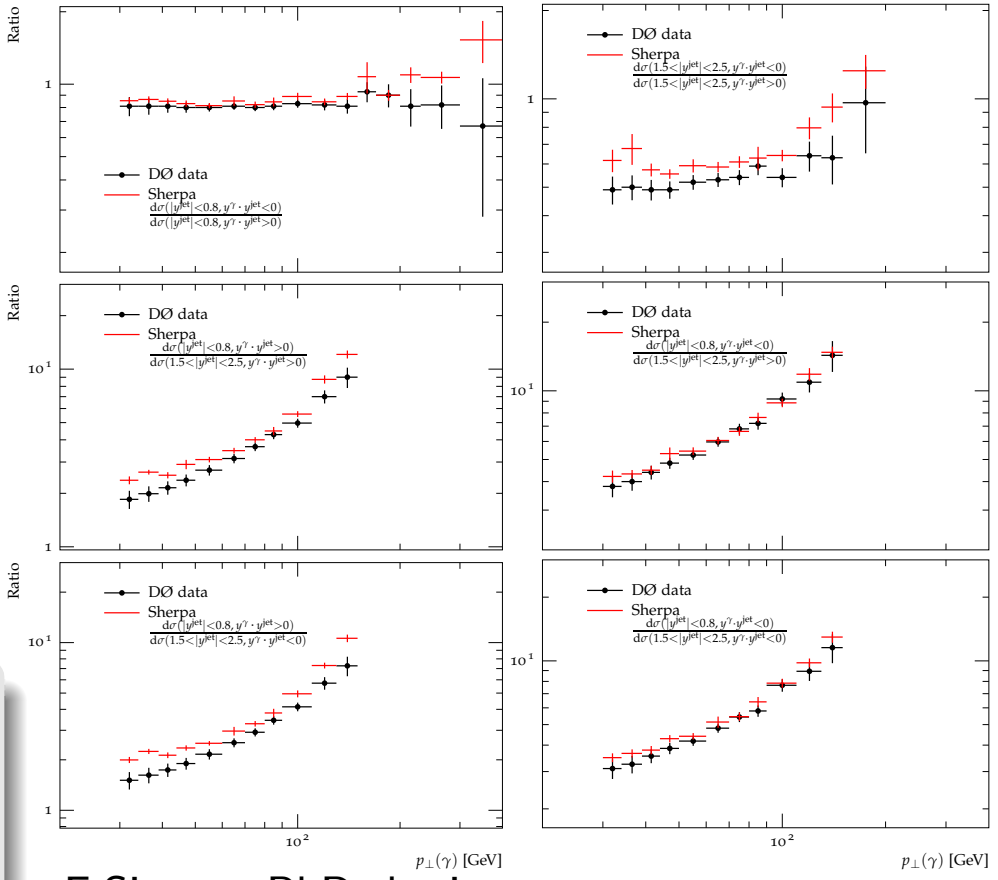
in regions of jet rapidity/orientation
scaled by 5, 1, 0.3 and 0.1 top to bottom



F.Siegert, PhD thesis

Ratio of photon p_T spectra PLB666(2008)435

compare regions of jet rapidity/orientation



F.Siegert, PhD thesis

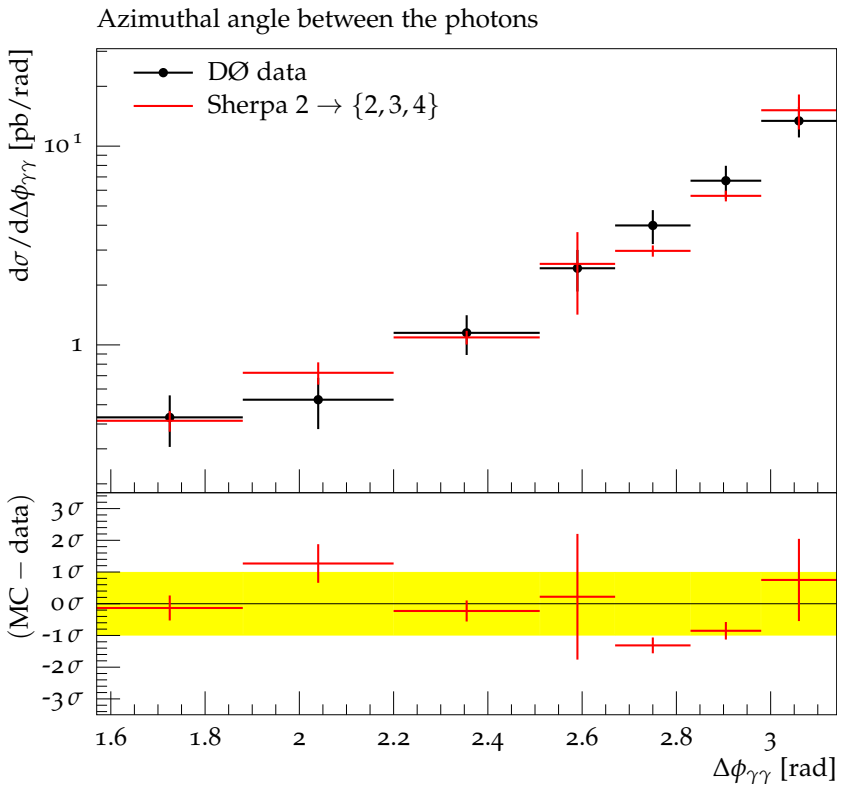
"Democratic" model ZPC62(1994)311

- Treat partons and γ equally
- Combine ME of various parton/γ multiplicity with
- Interleaved QCD⊕QED PS

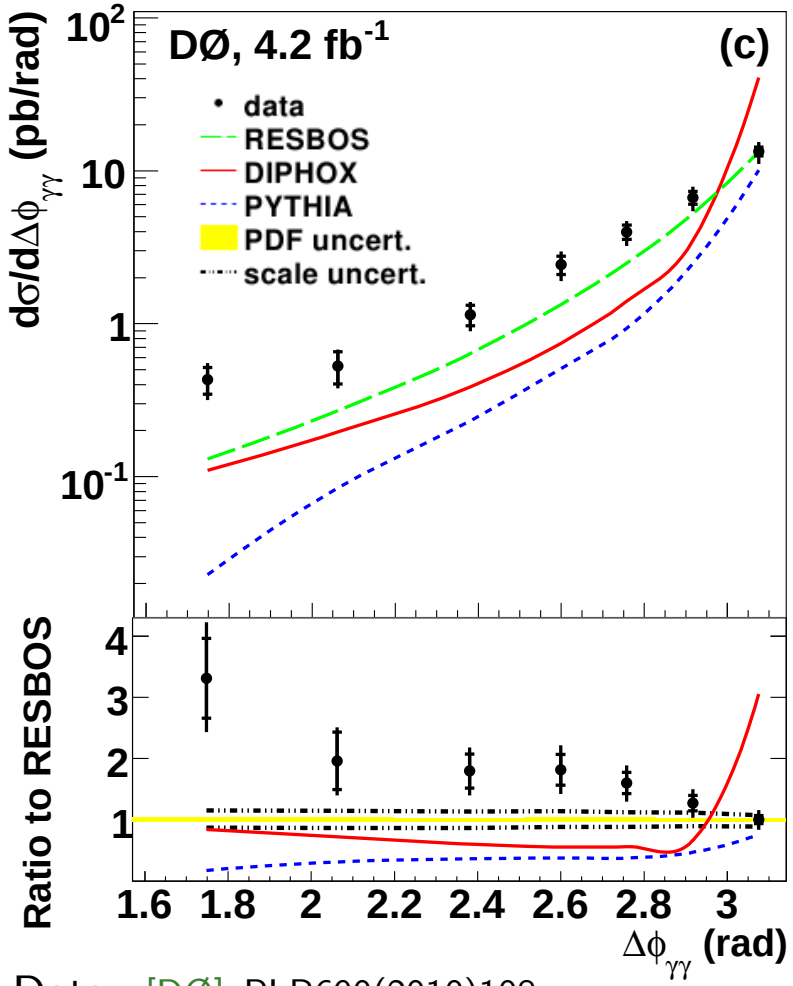
[Schumann,Siegert,SH] PRD81(2010)034026

Tree-level ME⊗PS at work: Di-photon production

$E_T^{\gamma 1} > 21$ GeV, $E_T^{\gamma 1} > 20$ GeV,
$|\eta^\gamma| < 0.9$, $E_T^{R=0.4} - E_T^\gamma < 2.5$ GeV



[Schumann,Siegert,SH] PRD81(2010)034026

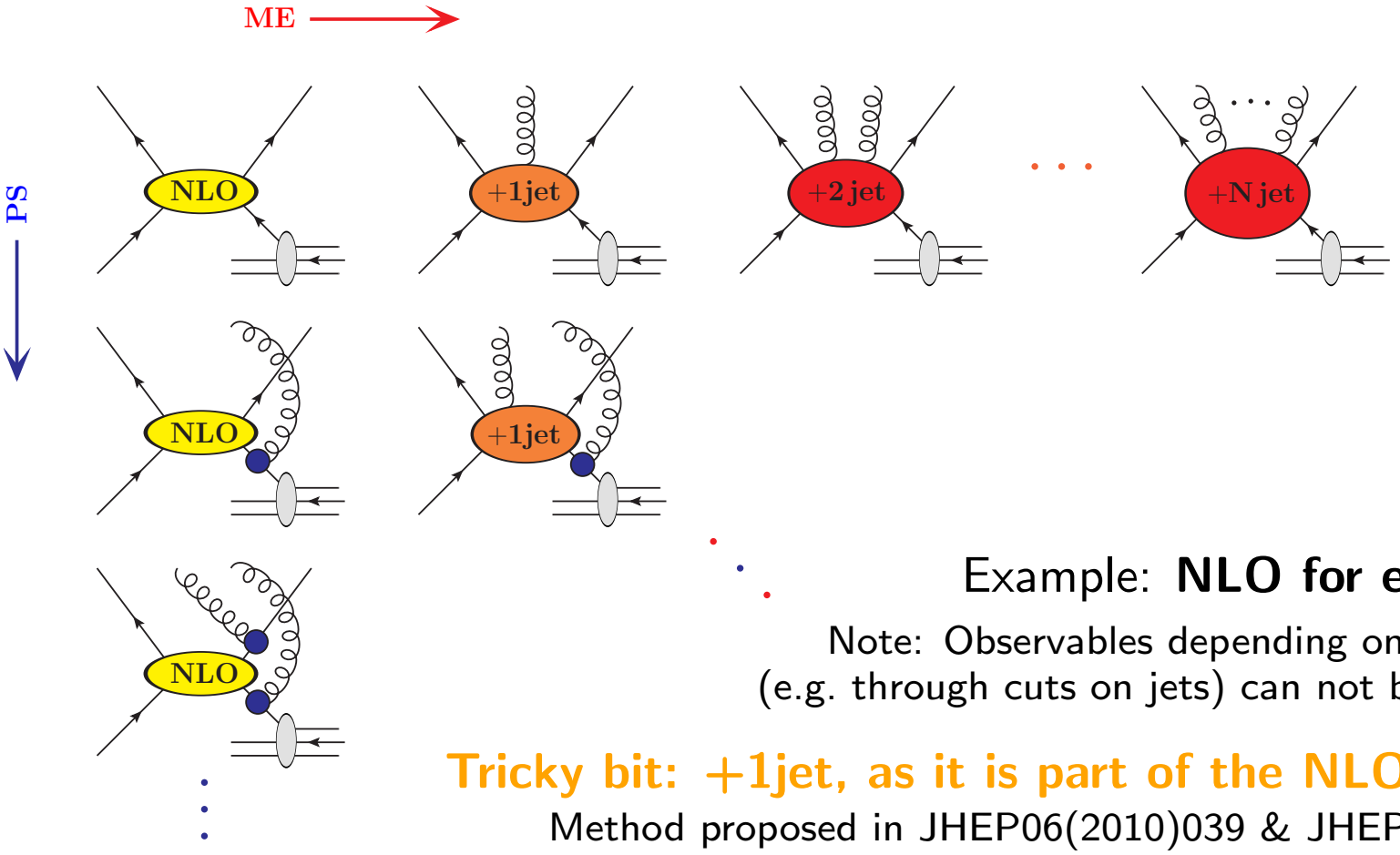


Data: [DØ] PLB690(2010)108

SHERPA prediction: Merged $2 \to \{2+3+4\}$-jet/γ plus $gg \to \gamma\gamma box$

Going NLO - The MENLOPS idea

Promoting the simulation to NLO accuracy for the core is currently cutting edge



Example: **NLO for $e^{\pm}q \to e^{\pm}q$**

Note: Observables depending on higher multi (e.g. through cuts on jets) can not be improved !

Tricky bit: +1jet, as it is part of the NLO core, too

Method proposed in JHEP06(2010)039 & JHEP08(2011)123

Note that we don't merge NLO with higher-multi NLO **yet** !

NLO calculations and parton-shower Monte Carlo

Anatomy of NLO calculations when using subtraction

$$\sigma^{NLO} = \int \mathrm{d}\Phi_B \left(\mathrm{B} + \tilde{\mathrm{V}}\right) + \int \mathrm{d}\Phi_R \, \mathrm{R} = \int \mathrm{d}\Phi_B \left[\left(\mathrm{B} + \tilde{\mathrm{V}} + \mathrm{I}\right) + \int \mathrm{d}\Phi_{R|B} \, (\mathrm{R} - \mathrm{S})\right]$$

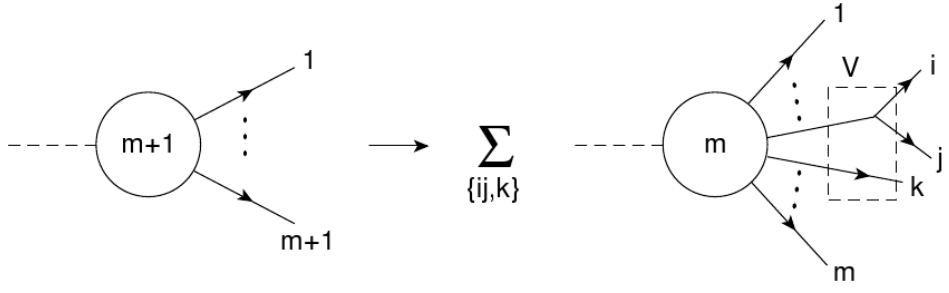
S - subtraction term constructed such that IR singularities in R are removed

I - integrated subtraction term locally (in Φ_B) compensates $\mathrm{S} \to 0 \overset{!}{=} \mathrm{I} - \int \mathrm{d}\Phi_{R|B} \, \mathrm{S}$

Consider for example Catani-Seymour dipole subtraction

[Catani,Seymour] NPB485(1997)291, [Catani,Dittmaier,Seymour,Trocsanyi] NPB627(2002)189

Schematically: $\mathrm{S} \to \sum_{\{i,j\}} \sum_{k \neq i,j} D_{ij,k}$



$D_{ij,k} = \mathrm{B}_{\tilde{ij},k} \otimes V_{ij,k}$ - dipole terms

$V_{ij,k}$ - process-independent dipole functions

Φ_R-kinematics different from $\Phi_B \to$ impossible to combine into one MC event

Solutions: MC@NLO [Frixione,Webber] JHEP06(2002)029 & POWHEG [Nason] JHEP11(2004)040

On next slides intro to:
- **Singular limits of S → modern parton showers**
- **Adding NLO back in → POWHEG & MC@NLO**

NLO calculations and parton-shower Monte Carlo

Need to be specific now: $\{\vec{a}\}$ means set of partons flavors $\{\vec{f}\}$, momenta $\{\vec{p}\}$

$\rightarrow$ **Real-emission contribution to NLO cross section**

$$\mathrm{d}\sigma_R(\{\vec{p}\}) = \sum_{\{\vec{f}\}} \mathrm{d}\sigma_R(\{\vec{a}\}) \qquad \mathrm{d}\sigma_R(\{\vec{a}\}) = \mathrm{d}\Phi_R(\{\vec{p}\})\,\mathrm{R}(\{\vec{a}\})$$

where $\mathrm{R}(\{\vec{a}\}) = \mathcal{L}(\{\vec{a}\})\,\mathcal{R}(\{\vec{a}\})$ and $\mathcal{L}(\{\vec{a}\};\mu^2) = x_1 f_{f_1}(x_1,\mu^2)\; x_2 f_{f_2}(x_2,\mu^2)$
$\mathrm{d}\Phi_R$ contains initial-state phase space $\mathrm{d}\log x_1\,\mathrm{d}\log x_2$
$\mathcal{R}(\{\vec{a}\}) = |\mathcal{M}_R|^2(\{\vec{a}\})/[F(\{\vec{a}\})S(\{\vec{f}\})]$ with symmetry factor S, flux F

Similar formulas for Born-level term $\mathrm{B}(\{\vec{a}\})$ one parton less, of course

Assume generalized "dipole terms", such that think of $D_{ij,k}$ on previous slide

$$\mathcal{R}(\{\vec{a}\}) \overset{\text{soft/collinear}}{\longrightarrow} \sum_{\{i,j\}} \sum_{k\neq i,j} \mathcal{D}_{ij,k}(\{\vec{a}\})$$

Define partition of real-emission term $\mathcal{R}(\{\vec{a}\}) = \sum_{\{i,j\}} \sum_{k\neq i,j} \mathcal{R}_{ij,k}(\{\vec{a}\})$

$$\mathcal{R}_{ij,k}(\{\vec{a}\}) := \rho_{ij,k}(\{\vec{a}\})\,\mathcal{R}(\{\vec{a}\})\,, \quad \text{where} \quad \rho_{ij,k}(\{\vec{a}\}) = \frac{\mathcal{D}_{ij,k}(\{\vec{a}\})}{\sum_{\{m,n\}} \sum_{l\neq m,n} \mathcal{D}_{mn,l}(\{\vec{a}\})}$$

Note: Holds throughout the phase space ! most important formula in the talk

NLO calculations and parton-shower Monte Carlo

$\mathcal{D}_{ij,k}(\{\vec{a}\})$ defines parton maps think of Catani-Seymour dipoles

$$b_{ij,k}(\{\vec{a}\}) = \begin{cases} \{\vec{f}\} \setminus \{f_i, f_j\} \cup \{f_{\widetilde{ij}}\} \\ \{\vec{p}\} \to \{\vec{\tilde{p}}\} \end{cases} \quad \leftrightarrow \quad r_{\widetilde{ij},\tilde{k}}(f_i, \Phi_{R|B}\,; \{\vec{a}\}) = \begin{cases} \{\vec{f}\} \setminus \{f_{\widetilde{ij}}\} \cup \{f_i, f_j\} \\ \{\vec{\tilde{p}}\} \to \{\vec{p}\} \end{cases}$$

- $b_{ij,k}$ converts real-emission configuration to Born-level
- $r_{\widetilde{ij},\tilde{k}}$ converts Born-level to real-emission needs extra flavor & phase space

Trivially factorize real-emission term into Born and radiative contribution

$$\mathrm{d}\sigma_R(\{\vec{a}\}) = \sum_{\{i,j\}} \sum_{k \neq i,j} \mathrm{d}\sigma_B(b_{ij,k}(\{\vec{a}\}))\, \mathrm{dP}_{ij,k}(\{\vec{a}\})$$

differential emission probability is $\mathrm{dP}_{ij,k}(\{\vec{a}\}) = \mathrm{d}\Phi^{ij,k}_{R|B}(\{\vec{p}\})\, \dfrac{\mathrm{R}_{ij,k}(\{\vec{a}\})}{\mathrm{B}(b_{ij,k}(\{\vec{a}\}))}$

Subtraction algorithms predict $\mathrm{dP}_{ij,k}$ in the soft/collinear limits via

$$\mathcal{D}_{ij,k}(\{\vec{a}\}) \overset{\text{soft/collinear}}{\longrightarrow} \frac{S(b_{ij,k}(\{\vec{f}\}))}{S(\{\vec{f}\})} \frac{1}{2\, p_i p_j}\, 8\pi\, \alpha_s\, \mathcal{B}(b_{ij,k}(\{\vec{a}\})) \otimes V_{ij,k}(p_i, p_j, p_k)\ ,$$

Note the symmetry factors $\leftrightarrow$ factorization of invariant ME, not of specific process

$\otimes \to$ spin & color-correlations between $\mathcal{B}$ and V

NLO calculations and parton-shower Monte Carlo

Now make an approximation replace correlated with uncorrelated dipole kernel

$$\mathcal{B}(b_{ij,k}(\{\vec{a}\})) \otimes V_{ij,k}(p_i, p_j, p_k) \to \mathcal{B}(b_{ij,k}(\{\vec{a}\}))\, \mathcal{K}_{ij,k}(p_i, p_j, p_k)$$

Parametrize radiative phase space: $\mathrm{d}\Phi^{ij,k}_{R|B}(\{\vec{p}\}) = \frac{1}{16\pi^2}\,\mathrm{d}t\,\mathrm{d}z\,\frac{\mathrm{d}\phi}{2\pi}\,J_{ij,k}(t,z,\phi)$

Assume phase space gets filled successively in t $\leftrightarrow$ partons can be distinguished

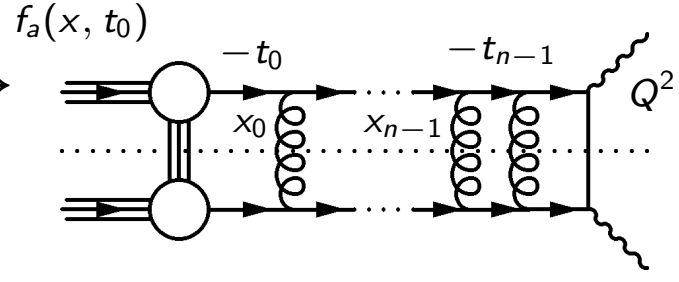
Must adapt symmetry factors: $\frac{S(b_{ij,k}(\{\vec{f}\}))}{S(\{\vec{f}\})} \to \frac{1}{S_{ij}} = \begin{cases} 1/2 & \text{if } i,j>2,\ b_i = b_j \\ 1 & \text{else} \end{cases}$

Combining everything gives differential radiation probability

$$\mathrm{dP}^{(\mathrm{PS})}_{ij,k}(\{\vec{a}\}) = \frac{\mathrm{d}t}{t}\,\mathrm{d}z\,\frac{\mathrm{d}\phi}{2\pi}\,\frac{\alpha_s}{2\pi}\,\frac{1}{S_{ij}}\,J_{ij,k}(t,z,\phi)\,\mathcal{K}_{ij,k}(t,z,\phi)\,\frac{\mathcal{L}(\{\vec{a}\};t)}{\mathcal{L}(b_{ij,k}(\{\vec{a}\});t)}$$

Iterate this equation for higher-multi ME
$\to$ ladder-like structure of amplitude squared
with strong ordering in scales $t_0 < \ldots < t_n$

$\leftrightarrow$



Factorization at any stage above Λ_{QCD}
can split emissions off ME one by one

Corrections induced by $\mathrm{dP}^{(\mathrm{PS})}_{ij,k}$ can be large and must be resummed

In inclusive case $t \in [0, \infty)$ divergences in $\mathcal{K}_{ij,k}$ cancel ε-poles in V $\to$ unitarity !

→ **No-emission probability** from Poisson statistics implementing unitarity constraint

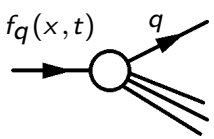
$$\mathcal{P}^{(\mathrm{PS})}_{\widetilde{ij},\tilde{k}}(t',t'';\{\vec{a}\}) = \exp\left\{ -\sum_{f_i=q,g} \int_{t'}^{t''} \int_{z_{\min}}^{z_{\max}} \int_0^{2\pi} \mathrm{dP}^{(\mathrm{PS})}_{ij,k}(r_{\widetilde{ij},\tilde{k}}(\{\vec{a}\})) \right\} .$$

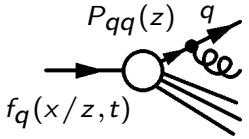
Note: $r_{\widetilde{ij}\tilde{k}}$ implicitly and uniquely defined by subtraction scheme, i.e. $\mathcal{K}_{ij,k}$

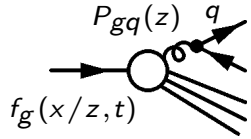
Assume IF-splitting → Lumi ratio $\frac{x}{z} f_{f_i}(\frac{x}{z},t)/x f_{f_{\widetilde{ij}}}(x,t)$, symmetry factor 1

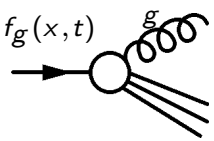
$$\frac{\partial \log \mathcal{P}^{(\mathrm{PS})}_{\widetilde{ij},\tilde{k}}(t,t';\{\vec{a}\})}{\partial \log(t/\mu^2)} = \int_x^{z_{\max}} \frac{\mathrm{d}z}{z} \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi} \sum_{f_i=q,g} \frac{\alpha_s}{2\pi} J_{ij,k}(t,z,\phi)\, \mathcal{K}_{ij,k}(t,z,\phi)\, \frac{f_{f_i}(\frac{x}{z},t)}{f_{f_{\widetilde{ij}}}(x,t)}$$

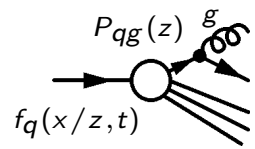
Voilà, the DGLAP equation ! imagine $J_{ij,k}(t,z,\phi)\mathcal{K}_{ij,k}(t,z,\phi) \to P_{i\,\widetilde{ij}}(z)$

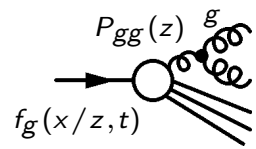
$$\frac{\mathrm{d}}{\mathrm{d}\log(t/\mu^2)}$$ 

$$= \int_x^1 \frac{\mathrm{d}z}{z}\frac{\alpha_s}{2\pi}$$ 

$$+ \int_x^1 \frac{\mathrm{d}z}{z}\frac{\alpha_s}{2\pi}$$ 

$$\frac{\mathrm{d}}{\mathrm{d}\log(t/\mu^2)}$$ 

$$= \sum_{i=1}^{2n_f} \int_x^1 \frac{\mathrm{d}z}{z}\frac{\alpha_s}{2\pi}$$ 

$$+ \int_x^1 \frac{\mathrm{d}z}{z}\frac{\alpha_s}{2\pi}$$ 

NLO calculations and parton-shower Monte Carlo

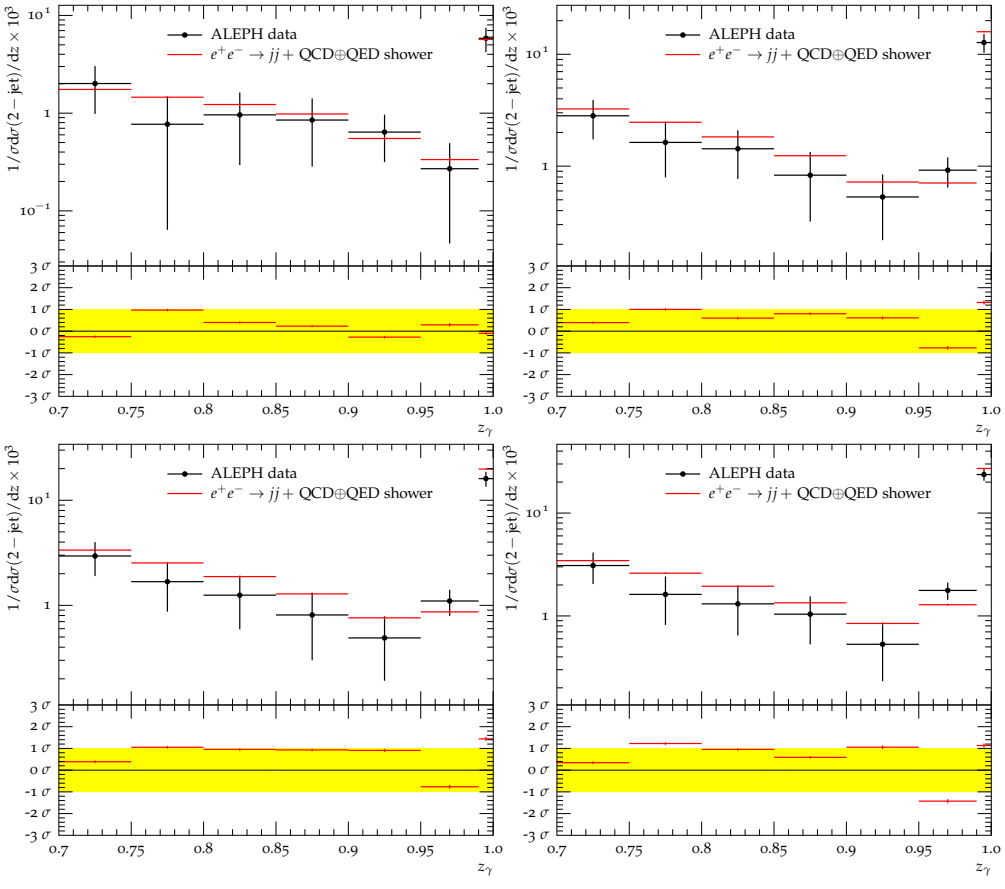
The Monte-Carlo implementation of this is a modern parton shower:

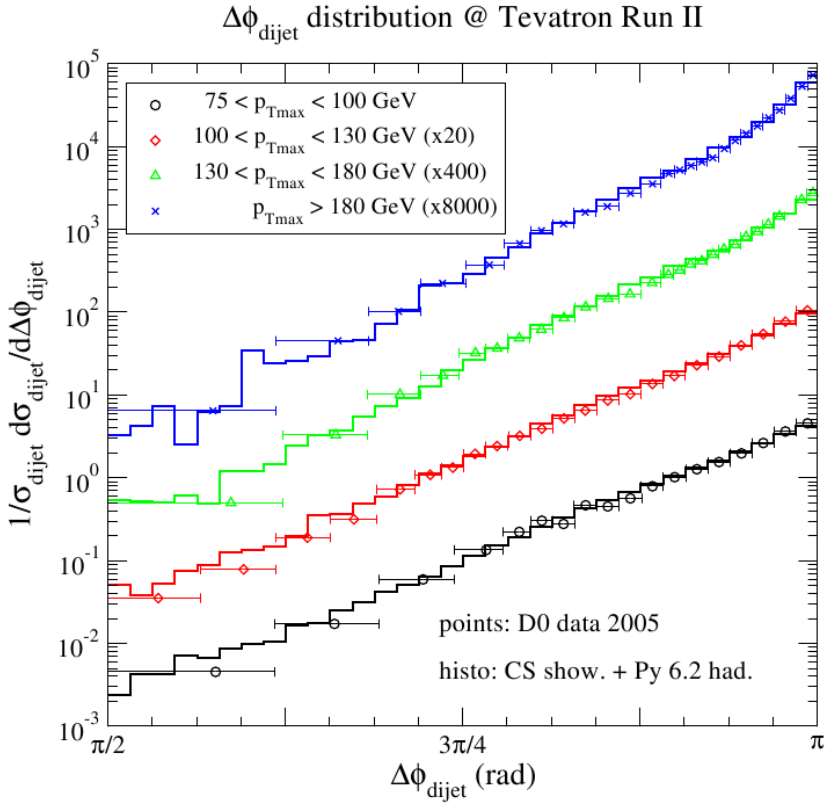
- Generate emission scale t from parton a at t' via

$$1 - \mathcal{P}^{(PS)}_{\widetilde{ij},\tilde{k}}(t, t'; \{\vec{a}\}) \stackrel{!}{=} \# \,, \qquad \text{where} \qquad \# \in [0,1] \text{ random}$$

- Generate splitting variable z, angle ϕ and flavor f_i according to $\mathrm{dP}^{(\mathrm{PS})}_{ij,k}(\{\vec{a}\})$

γ-fragmentation function PRD81(2010)034026





[Schumann,Krauss] JHEP03(2008)038

NLO calculations and parton-shower Monte Carlo

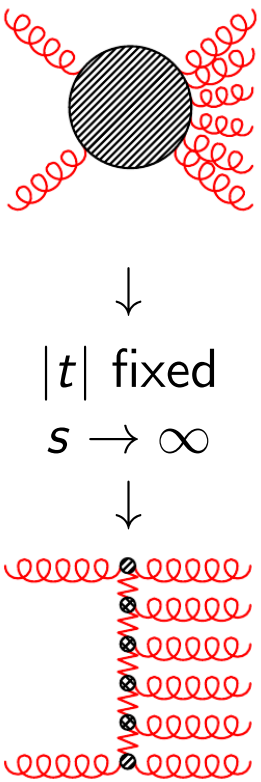
Aside: Catani-Seymour dipole shower compared to High Energy Jets (HEJ)

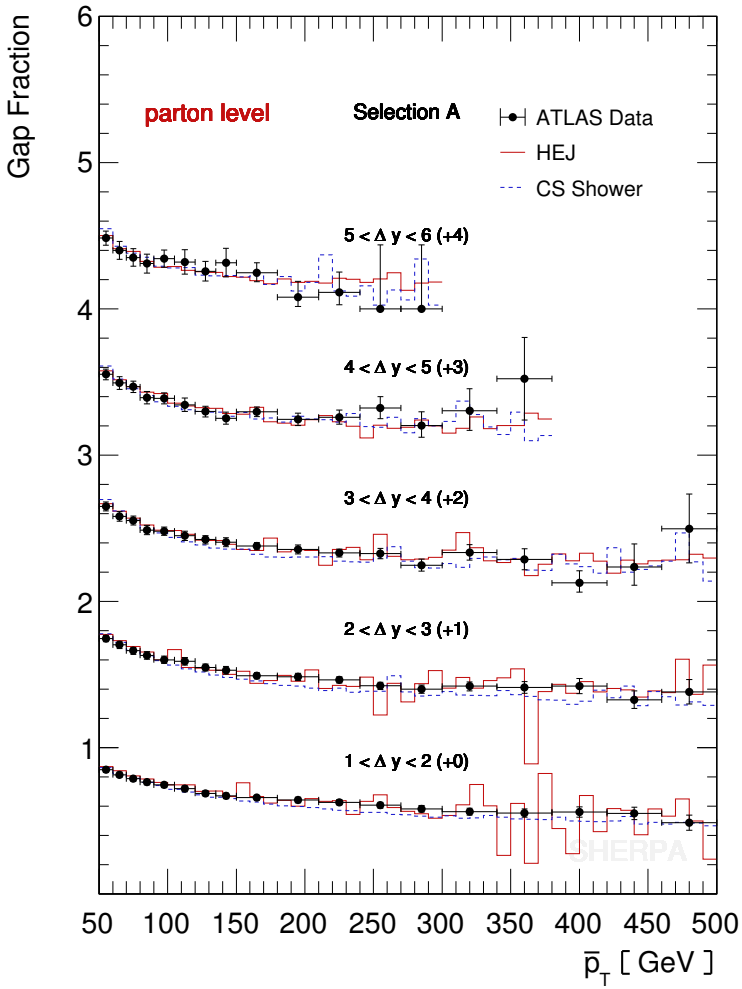
HEJ → All-order corrections to multi-jet rates from high-energy limit

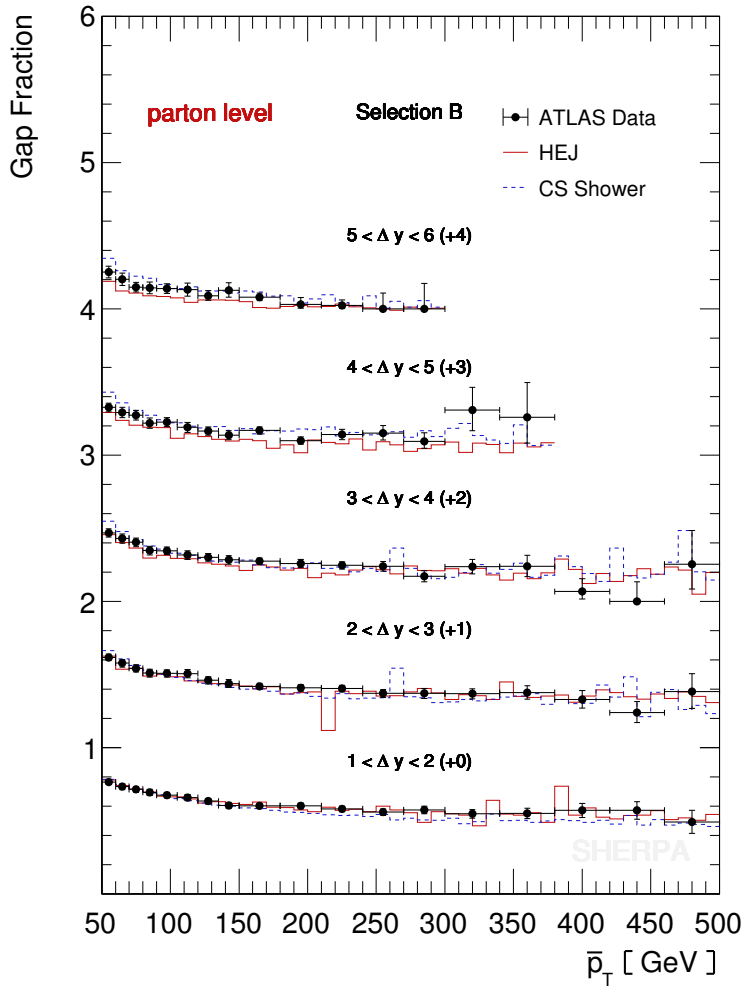
[Andersen,Smillie]

JHEP 06(2011)010

JHEP 01(2010)039







Resummation only. Exp. data: [Atlas] arXiv:1107.1641 [hep-ph]

The POWHEG method

Recover NLO-accurate radiation pattern in PS through correction weight

[Nason] JHEP11(2004)040 [Frixione,Nason,Oleari] JHEP11(2007)070

$$w_{ij,k}(\{\vec{a}\}) = \mathrm{dP}_{ij,k}(\{\vec{a}\}) \,/\, \mathrm{dP}^{(\mathrm{PS})}_{ij,k}(\{\vec{a}\})$$

Easy to automate in general-purpose Monte-Carlo, all input is tree-level only

Approximate "seed cross section" using local K-factor $\bar{\mathrm{B}}/\mathrm{B}$

$$\frac{\bar{\mathrm{B}}(\{\vec{a}\})}{\mathrm{B}(\{\vec{a}\})} = 1 + \frac{\tilde{\mathrm{V}}(\{\vec{a}\}) + \mathrm{I}(\{\vec{a}\})}{\mathrm{B}(\{\vec{a}\})} + \sum_{\{\tilde{ij},\tilde{k}\}} \sum_{f_i=q,g} \int \mathrm{d}\Phi^{ij,k}_{R|B} \frac{\mathrm{R}_{ij,k}(r_{\tilde{ij},\tilde{k}}(\{\vec{a}\})) - \mathrm{S}_{ij,k}(r_{\tilde{ij},\tilde{k}}(\{\vec{a}\}))}{\mathrm{B}(\{\vec{a}\})}$$

Note: Implies wrong dependence of observables on final-state momenta $\{\vec{p}\}$ → resolved by PS

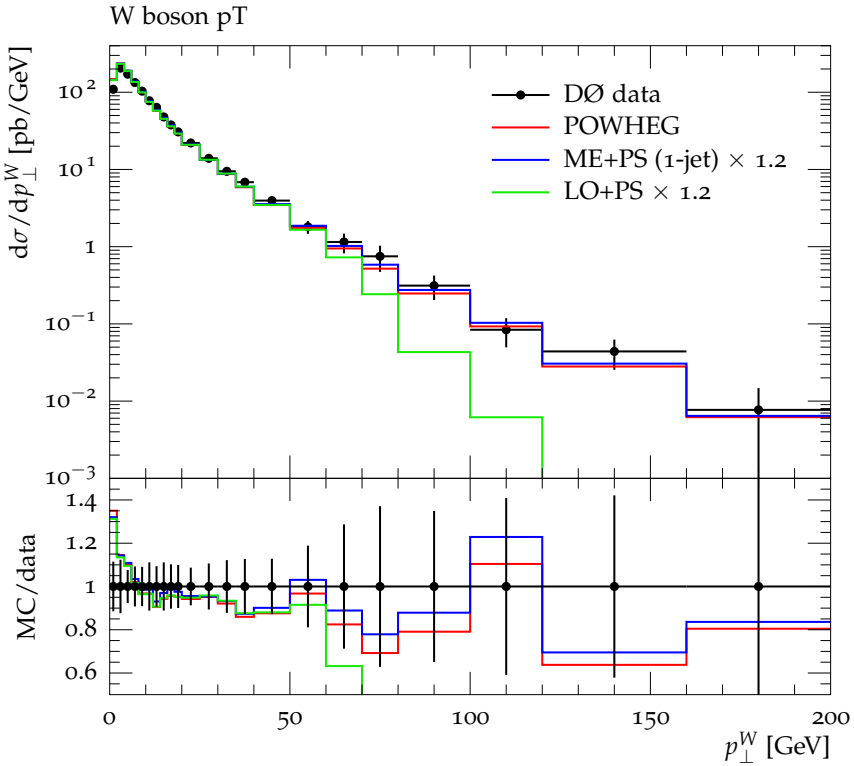
Combine ME-correction and local K-factor → observable O to $\mathcal{O}(\alpha_s)$ from

$$\begin{aligned}
\langle O \rangle^{(\mathrm{POWHEG})} = \sum_{\{\vec{f}\}} \int \mathrm{d}\Phi_B(\{\vec{p}\})\, \bar{\mathrm{B}}(\{\vec{a}\}) \Bigg[& \mathcal{P}(t_0, \mu^2; \{\vec{a}\})\, O(\{\vec{p}\}) \\
& + \sum_{\{\tilde{ij},\tilde{k}\}} \sum_{f_i=q,g} \frac{1}{16\pi^2} \int_{t_0}^{\mu^2} \mathrm{d}t \int_{z_{\min}}^{z_{\max}} \mathrm{d}z \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi}\, J_{ij,k}(t, z, \phi) \\
& \quad \times \frac{1}{S_{ij}} \frac{S(r_{\tilde{ij},\tilde{k}}(\{\vec{f}\}))}{S(\{\vec{f}\})} \frac{\mathrm{R}_{ij,k}(r_{\tilde{ij},\tilde{k}}(\{\vec{a}\}))}{\mathrm{B}(\{\vec{a}\})}\, \mathcal{P}(t, \mu^2; \{\vec{a}\})\, O(r_{\tilde{ij},\tilde{k}}(\{\vec{p}\}))\Bigg]
\end{aligned}$$

POWHEG predictions from SHERPA

W-boson p_T [DØ] PLB513(2001)292

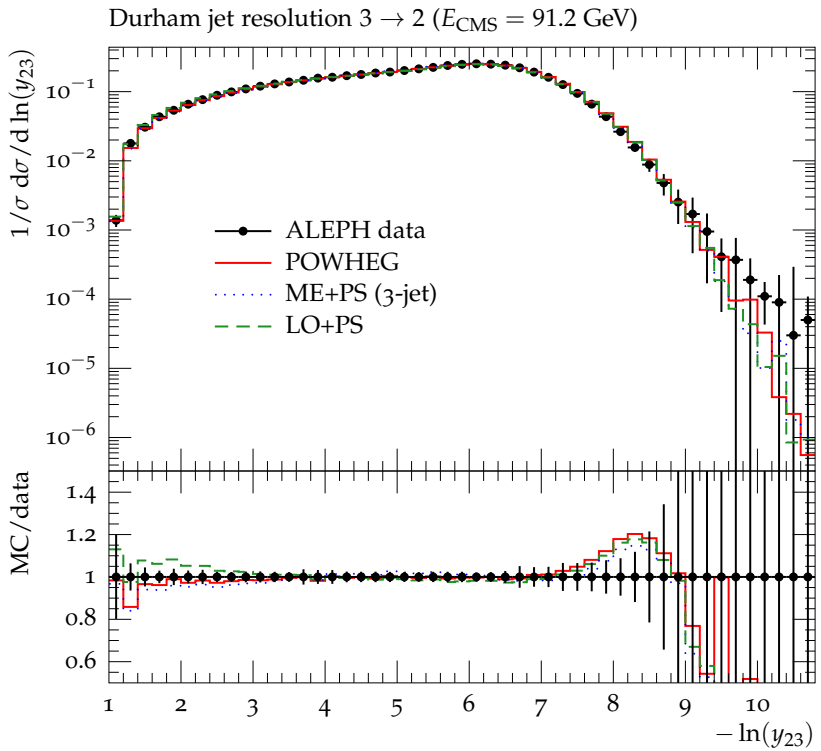
$pp \to l\nu$+jets @ $E_{\mathrm{cms}} = 1.8$ TeV



[Krauss,Schönherr,Siegert,SH] JHEP04(2011)024

Jet resolution [ALEPH] EPJC35(2004)457

$e^+e^- \to$hadrons @ $E_{\mathrm{cms}} = 91.2$ GeV

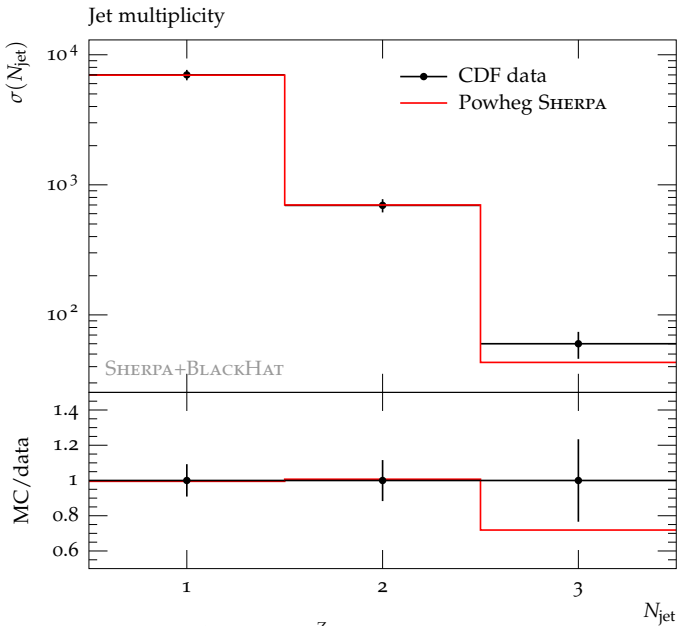


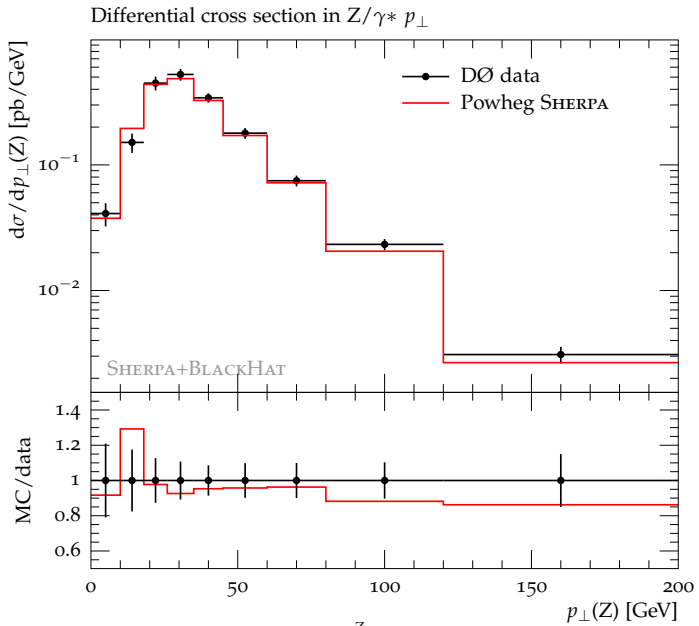
[Krauss,Schönherr,Siegert,SH] JHEP04(2011)024

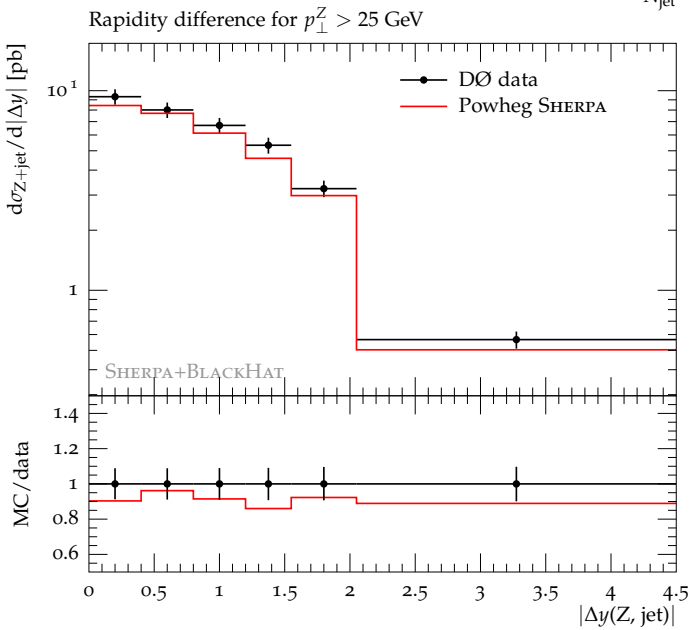
ME⊗PS and POWHEG agree in shape
Difference is **global** K-factor

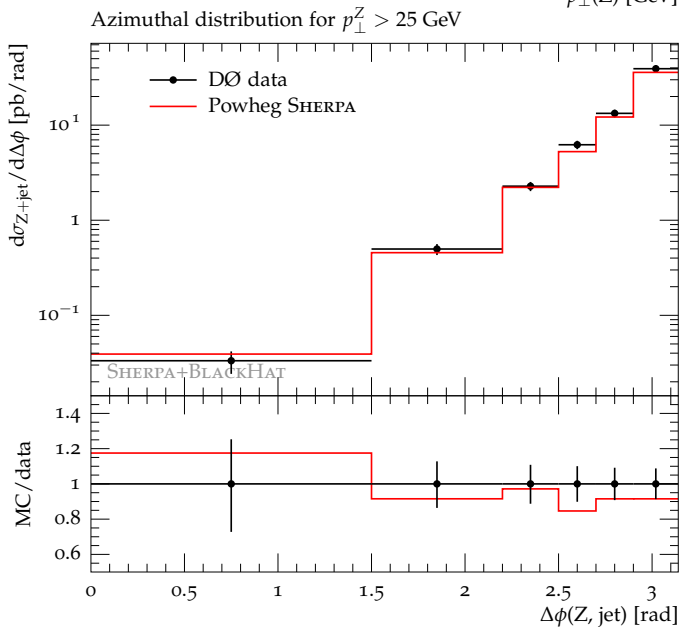
Higher-order corrections do not always favour harder emissions

POWHEG predictions from SHERPA









[Krauss,Schönherr,Siegert,SH] unpublished

Large-N_C only

SHERPA POWHEG vs. Tevatron data

[CDF] PRL100(2008)102001

[DØ] PLB669(2008)278

[DØ] PLB682(2010)370

The MC@NLO method

Modify POWHEG such that only parts of R exponentiated [Alioli et al.] JHEP07(2008)060

$$\begin{aligned}\langle O\rangle^{(\mathrm{mPOWHEG})} = \sum_{\{\vec{f}\}} \int \mathrm{d}\Phi_B(\{\vec{p}\})\, \bar{\mathrm{B}}^{(\mathrm{D})}(\{\vec{a}\}) \Bigg[& \mathcal{P}^{(\mathrm{D})}(t_0,\mu^2;\{\vec{a}\})\, O(\{\vec{p}\}) \\ & + \sum_{\{\tilde{ij},\tilde{k}\}} \sum_{f_i=q,g} \frac{1}{16\pi^2} \int_{t_0}^{\mu^2} \mathrm{d}t \int_{z_{\min}}^{z_{\max}} \mathrm{d}z \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi}\, J_{ij,k}(t,z,\phi) \\ & \times \frac{1}{S_{ij}} \frac{S(r_{\tilde{ij},\tilde{k}}(\{\vec{f}\}))}{S(\{\vec{f}\})} \frac{\mathrm{D}_{ij,k}(r_{\tilde{ij},\tilde{k}}(\{\vec{a}\}))}{\mathrm{B}(\{\vec{a}\})}\, \mathcal{P}^{(\mathrm{D})}(t,\mu^2;\{\vec{a}\})\, O(r_{\tilde{ij},\tilde{k}}(\{\vec{p}\}))\Bigg] \\ & + \sum_{\{\vec{f}\}} \int \mathrm{d}\Phi_R(\{\vec{p}\}) \Big[\mathrm{R}_{ij,k}(\{\vec{a}\}) - \mathrm{D}_{ij,k}(\{\vec{a}\})\Big]\, O(\{\vec{p}\})\end{aligned}$$

Seed cross sections change correspondingly as

$$\frac{\bar{\mathrm{B}}^{(\mathrm{D})}(\{\vec{a}\})}{\mathrm{B}(\{\vec{a}\})} = 1 + \frac{\tilde{\mathrm{V}}(\{\vec{a}\}) + \mathrm{I}(\{\vec{a}\})}{\mathrm{B}(\{\vec{a}\})} + \sum_{\{\tilde{ij},\tilde{k}\}} \sum_{f_i=q,g} \int \mathrm{d}\Phi^{ij,k}_{R|B} \frac{\mathrm{D}_{ij,k}(r_{\tilde{ij},\tilde{k}}(\{\vec{a}\})) - \mathrm{S}_{ij,k}(r_{\tilde{ij},\tilde{k}}(\{\vec{a}\}))}{\mathrm{B}(\{\vec{a}\})}$$

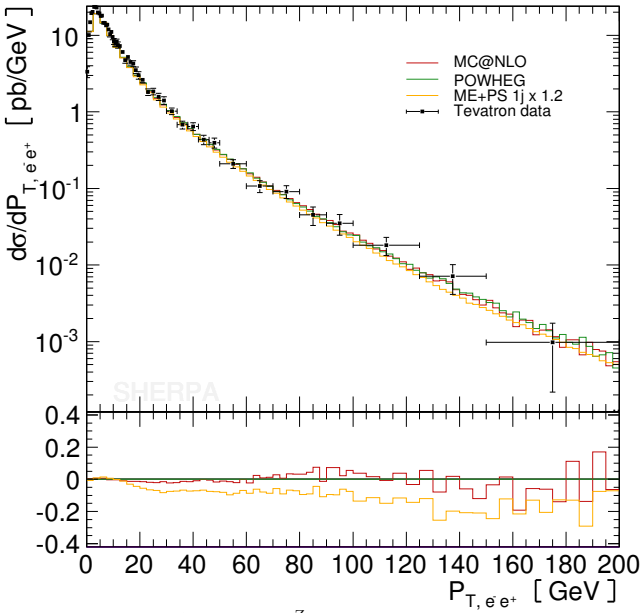
Great simplification if $\mathrm{D}_{ij,k} \to \mathrm{S}_{ij,k}$ recover POWHEG for $\mathrm{D}_{ij,k} \to \mathrm{R}_{ij,k}$

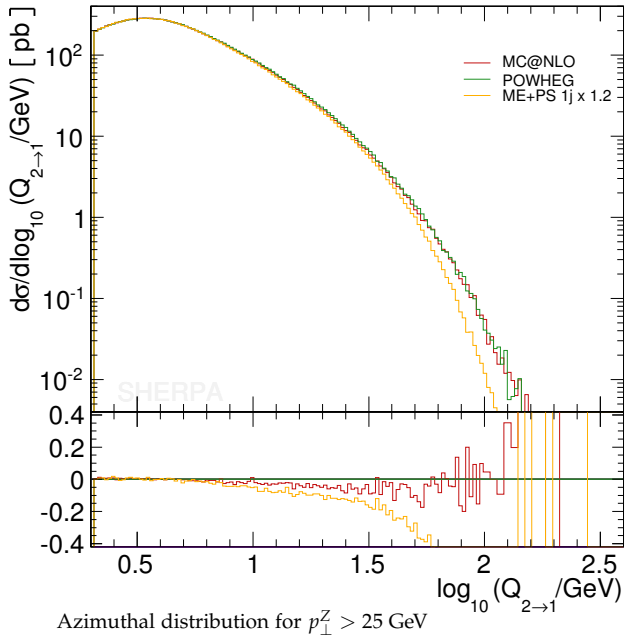
MC@NLO emerges as another special case when [Frixione,Webber] JHEP06(2002)029

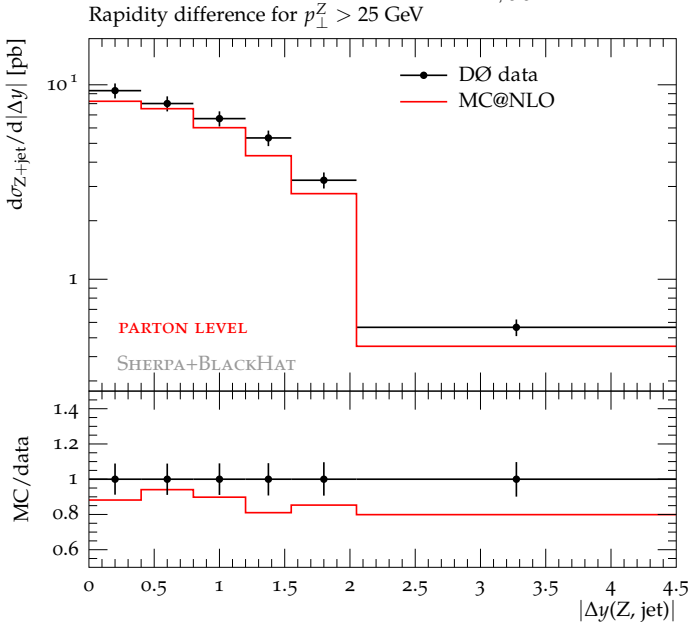
$\mathrm{S}_{ij,k}(\{\vec{a}\}) \to S(b_{ij,k}(\{\vec{f}\}))/S(\{\vec{f}\})\; 8\pi\,\alpha_s/2\, p_i p_j\, \mathrm{B}(b_{ij,k}(\{\vec{a}\}))\, \mathcal{K}_{ij,k}(p_i,p_j,p_k)$

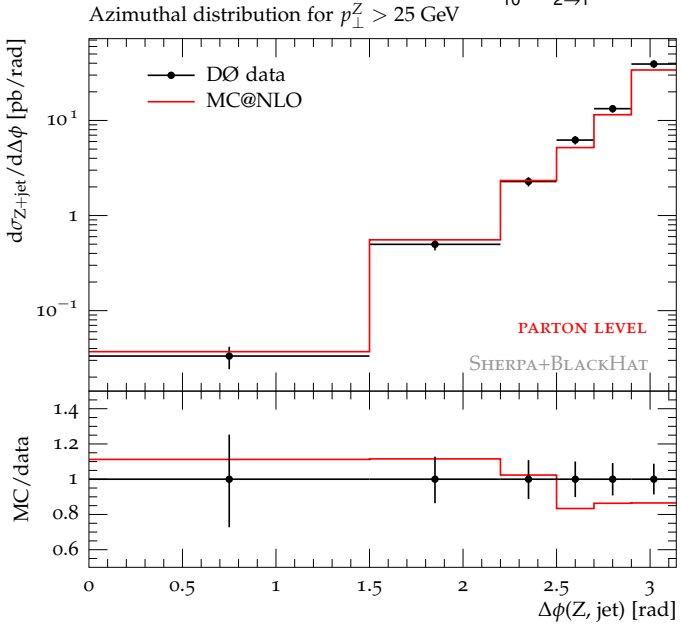
i.e. when the parton-shower approximation of R is used for subtraction

MC@NLO predictions from SHERPA









Preliminary!

SHERPA MC@NLO vs. Tevatron data

[CDF] PRL84(2000)845

[DØ] PLB682(2010)370

[Krauss,Schönherr,Siegert,SH] to appear

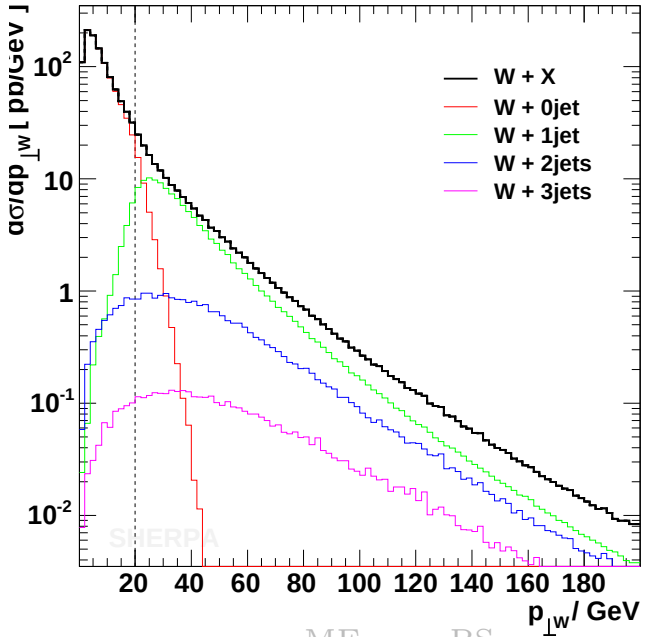
The ME⊗PS method

ME⊗PS idea: **Separate phase space into "hard" and "soft" region**

- Matrix elements populate hard domain
- Parton shower populates soft domain

need criterion to define "hard" & "soft" as above and below a certain cut value
→ **Jet criterion** Q e.g. k_T-jet measure

$\longleftrightarrow$



Formally: replace kernels in PS evolution with note that $\mathcal{K}_{ij,k} = \mathcal{K}^{\mathrm{ME}}_{ij,k} + \mathcal{K}^{\mathrm{PS}}_{ij,k}$

$$\mathcal{K}^{\mathrm{ME}}_{ij,k}(t,z,\phi) = \mathcal{K}_{ij,k}(t,z,\phi)\,\Theta\left[Q_{ij,k}(t,z,\phi) - Q_{\mathrm{cut}}\right] \quad \rightarrow \quad \mathcal{P}^{(\mathrm{PS})\,\mathrm{ME}}_{\tilde{ij},\tilde{k}}(t,t')$$
$$\mathcal{K}^{\mathrm{PS}}_{ij,k}(t,z,\phi) = \mathcal{K}_{ij,k}(t,z,\phi)\,\Theta\left[Q_{\mathrm{cut}} - Q_{ij,k}(t,z,\phi)\right] \quad \rightarrow \quad \mathcal{P}^{(\mathrm{PS})\,\mathrm{PS}}_{\tilde{ij},\tilde{k}}(t,t')$$

Reweight PS **emission** by $w_{ij,k}$ in ME domain only

Formally equivalent to POWHEG but **uncorrected no-emission rate** $\mathcal{P}^{(\mathrm{PS})}$

In contrast to POWHEG, ME⊗PS allows to approximate corrections beyond NLO!

Combining ME⊗PS and POWHEG/MC@NLO: MENLOPS

Slice the POWHEG/MC@NLO phase space in ME⊗PS-style

[Hamilton,Nason] JHEP06(2010)039, [Krauss,Schönherr,Siegert,SH] JHEP08(2011)123

$$\langle O\rangle^{(\mathrm{MENLOPS})} = \sum_{\{\vec{f}\}} \int \mathrm{d}\Phi_B(\{\vec{p}\})\,\bar{\mathrm{B}}(\{\vec{a}\}) \Bigg[\underbrace{\mathcal{P}(t_0,\mu^2;\{\vec{a}\})\,O(\{\vec{p}\})}_{\text{unresolved}}$$

$$+ \sum_{\{\widetilde{ij},\tilde{k}\}} \sum_{f_i=q,g} \frac{1}{16\pi^2} \int_{t_0}^{\mu^2} \mathrm{d}t \int_{z_{\min}}^{z_{\max}} \mathrm{d}z \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi}\, J_{ij,k}(t,z,\phi)\; O(r_{\widetilde{ij},\tilde{k}}(\{\vec{p}\}))$$

$$\times \frac{1}{S_{ij}} \frac{S(r_{\widetilde{ij},\tilde{k}}(\{\vec{f}\}))}{S(\{\vec{f}\})} \frac{\mathrm{R}_{ij,k}(r_{\widetilde{ij},\tilde{k}}(\{\vec{a}\}))}{\mathrm{B}(\{\vec{a}\})} \Bigg(\overbrace{\mathcal{P}(t,\mu^2;\{\vec{a}\})\,\Theta\left[Q_{\mathrm{cut}} - Q_{ij,k}(t,z,\phi)\right]}^{\text{resolved, PS domain}}$$

$$+ \underbrace{\mathcal{P}^{(\mathrm{PS})}(t,\mu^2;\{\vec{a}\})\,\Theta\left[Q_{ij,k}(t,z,\phi) - Q_{\mathrm{cut}}\right]}_{\text{resolved, ME domain}} \Bigg) \Bigg]$$

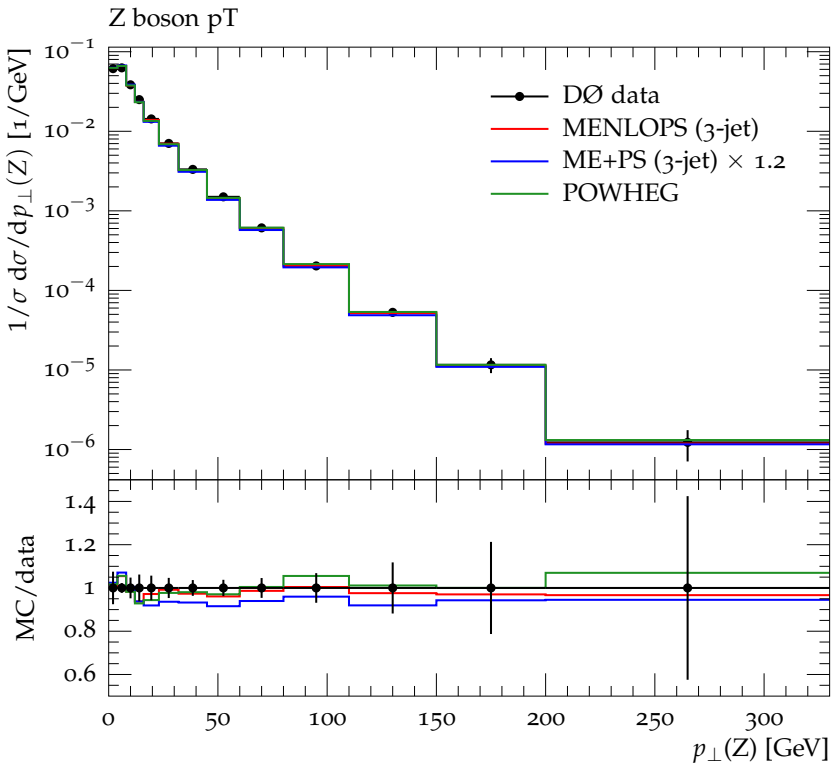
Note: Local K-factor $\bar{\mathrm{B}}/\mathrm{B}$ must be applied to ME⊗PS before merging

Overall characteristics

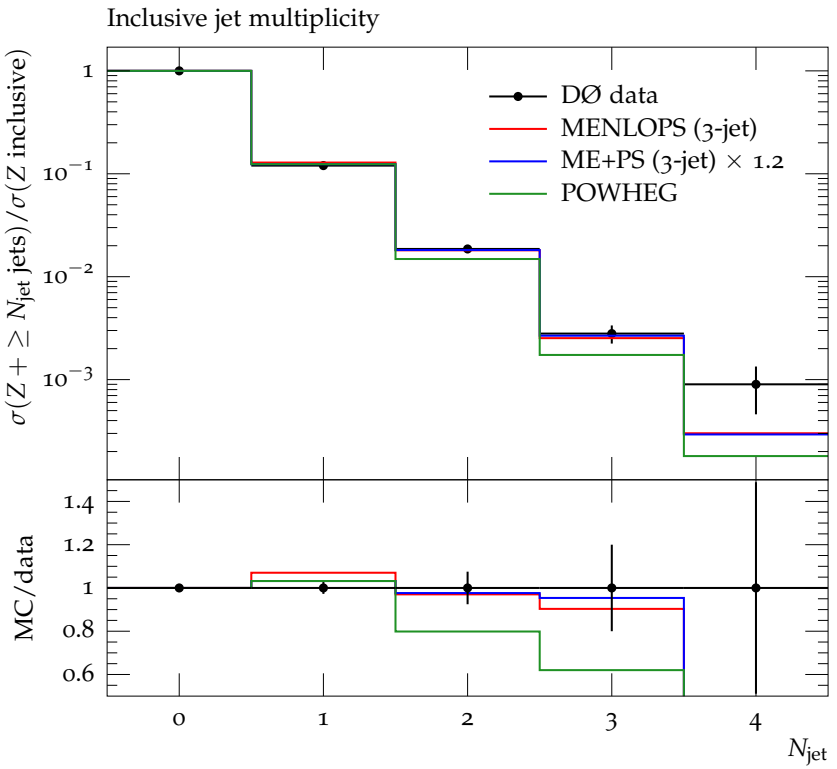
- Accuracy inherited from POWHEG/MC@NLO → stable rates for core process
- Higher-order tree-level via ME⊗PS → improved multi-jet predictions
- **Unitarity violation inherited from ME⊗PS** not necessarily bad, see later !

MENLOPS for Z+jets

Z-boson p_T [DØ] PLB693(2010)522



Jet multiplicity [DØ] PLB658(2008)112

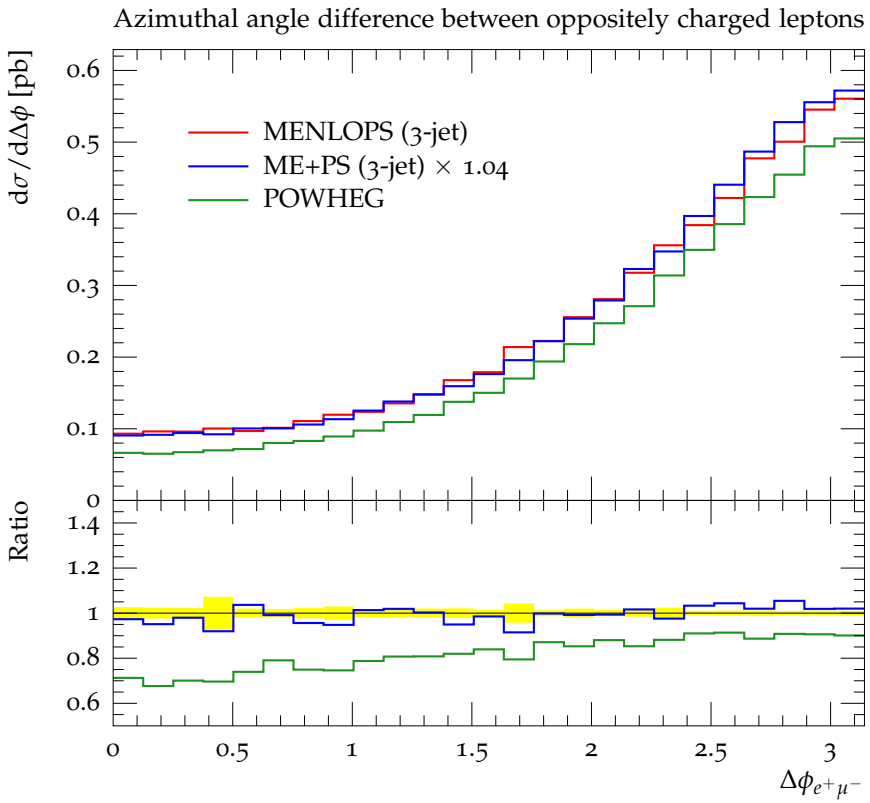


First Run II measurement using μ's
Data corrected to the particle level

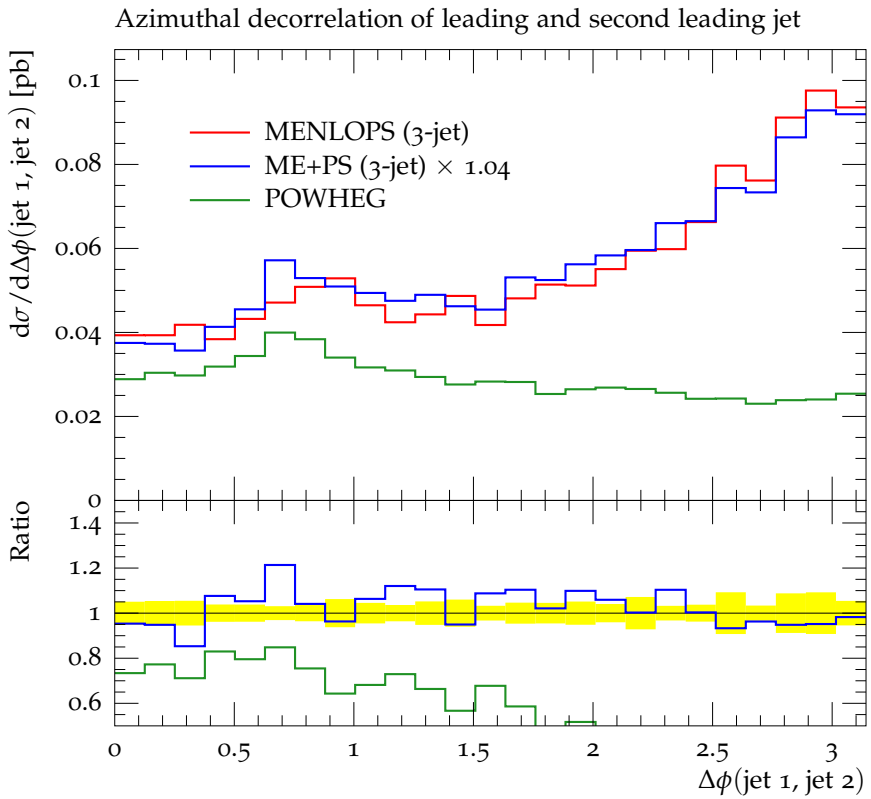
SHERPA prediction: $p\bar{p} \to \ell\ell$@NLO
$\oplus$ $p\bar{p} \to \ell\ell$+{1,2,3}-jets@LO

MENLOPS for W^+W^-+jets

Lepton-$\Delta\phi$ (LHC @ 14 TeV)



Jet-$\Delta\phi$ (LHC @ 14 TeV)



Large unitarity violations in both ME⊗PS and MENLOPS

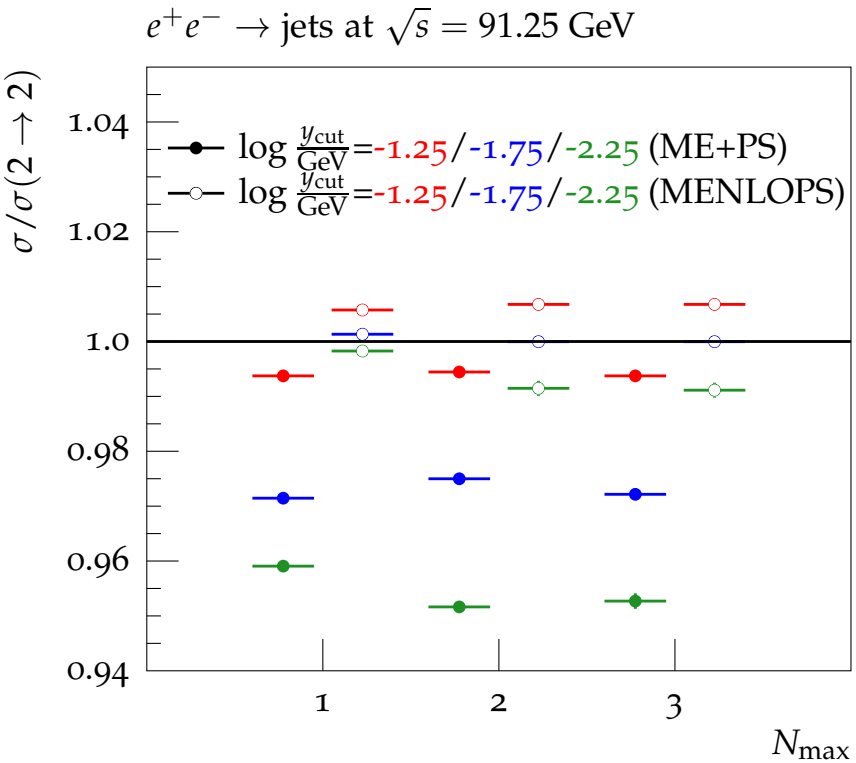
Follows from large W^+W^-+jets contribution, cf. PRD72(2005)034028, PLB683(2010)154

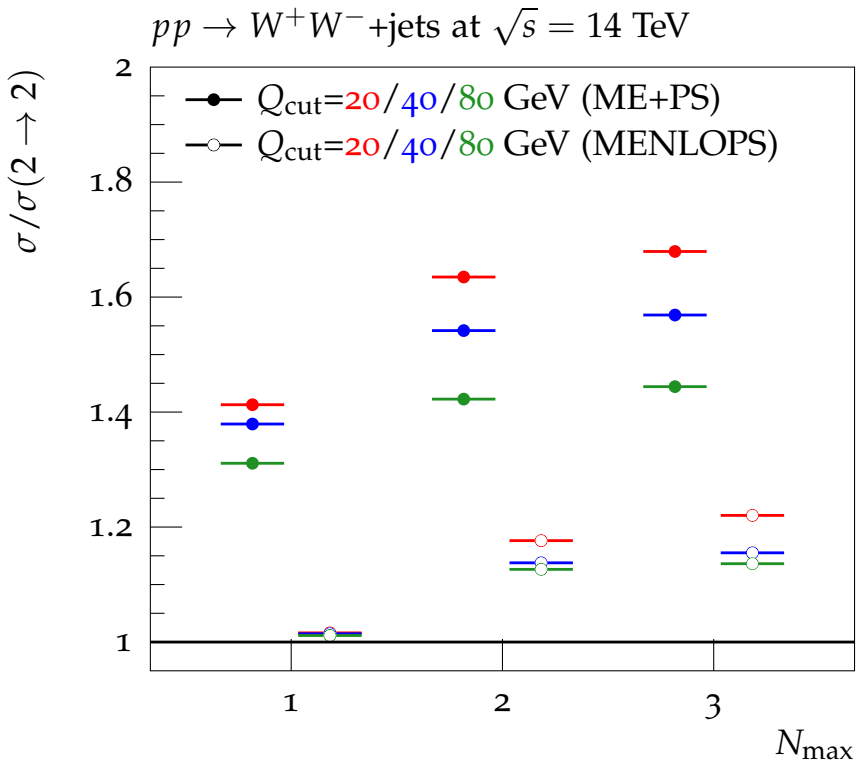
Note: **This is by no means a shortcoming of the method!**

Large phase space for 2 additional jets. $\Delta\phi_{jj}$ only LO_{1j}⊗LL in POWHEG

MENLOPS inclusive cross sections

ME⊗PS/ MENLOPS cross sections compared to $2 \to 2$ LO / NLO





Large unitarity violations in ME⊗PS reduced by MENLOPS

remaining discrepancies to POWHEG potentially indicative of higher-order corrections

Summary

Monte-Carlo event generators have improved:

- Reduced uncertainty due to fully/partially automated NLO MEs
- Largely reduced systematics with ME⊗PS, POWHEG & MC@NLO
- Reliable predictions for LEP, HERA, Tevatron and LHC
- ME⊗PS and MENLOPS fully automated, to become new standard

More and more higher-order pQCD built into MC event generators!

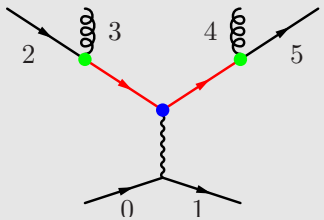
Models only where necessary mostly for non-perturbative aspects

Implementing POWHEG: The local K-factor

Integration of $\bar{\mathrm{B}}(\Phi_B)$ proceeds in two steps ...

Step I: The Born phase space via recycling

Standard phase-space generator, e.g. single channels from NPB9(1969)568
VEGAS-refined CLNS-80/447(1980) and combined in multi-channel CPC83(1994)141

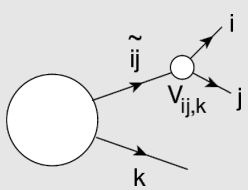


$\Leftrightarrow$

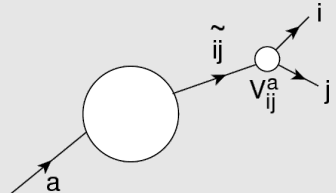
$$D_{iso}(23,45) \otimes P_0(23) \otimes P_0(45) \otimes D_{iso}(2,3) \otimes D_{iso}(4,5)$$

Step II: The real-emission phase space new

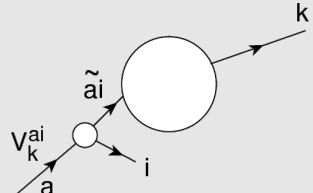
Extra emission generator (EEG) produces additional parton starting from Φ_B
Kinematics according to CS dipole terms NPB485(1997)291, NPB627(2002)189



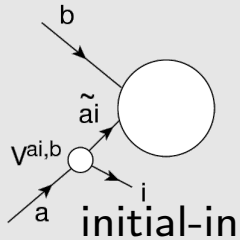
final-final



final-initial



initial-final



initial-initial

Implementing POWHEG: The local K-factor

Extra emission generator (EEG)
with multi-channeling over all dipole configurations

$\leftrightarrow$

Separate integration (sep)
of Born and real-emission kinematics with (modified) standard integrator

Process	σ [pb] (EEG)	σ [pb] (sep)	σ [pb] (LO)
$e^+e^- \to 2jets$	29449(19)	29454(18)	28381(18)
E_{cms}=91.2 GeV			
$e^+e^- \to 3jets$	9399(38)	9418(60)	7724(21)
as above, $y_{\mathrm{cut}} = 10^{-1.92}$			
$e^+e^- \to 4jets$	1377(14)	1357(21)	907(10)
as above, $y_{\mathrm{cut}} = 10^{-1.92}$			
$p\bar{p} \to e^-\bar{\nu}_e$	1331.7(5)	1332.2(4)	1098.6(3)
$E_{\mathrm{cms}} = 1.96$ TeV, CTEQ 6.6			
$p\bar{p} \to e^-\bar{\nu}_e + jet$	389.0(16)	390.6(17)	282.9(5)
as above, $k_T = 10$ GeV, $D = 0.7$			
$p\bar{p} \to e^-\bar{\nu}_e + 2jets$	104.2(7)	105.5(9)	73.9(2)
as above, $k_T = 10$ GeV, $D = 0.7$			

500k MC-points before cuts, no time limit, no error target
virtual part delivered by BlackHat [BlackHat] PRD78(2008)036003

Definition of a jet criterion

Catani-Seymour dipoles give good approximation of NLO real emission ME $\Rightarrow$
Use simplified dipole functions to define a distance measure

$$Q_{ij}^2 = 2\,p_i p_j \min_{k\neq i,j} \frac{2}{C_{i,j}^k + C_{j,i}^k}\,; \quad C_{i,j}^k = \begin{cases} \dfrac{p_i p_k}{(p_i+p_k)p_j} - \dfrac{m_i^2}{2\,p_i p_j} & \text{if } j = g \\ 1 & \text{else} \end{cases}$$

Initial state splittings: $C_{a,\,j}^k \to C_{(aj),\,j}^k$ splitting "towards" hard process

Why is this a sensible choice ?

Q must identify soft and collinear divergences of QCD MEs

Separate phase space by cutting on $Q \Leftrightarrow$ regularize tree-level MEs by cutting on Q

- Soft limit scale gluon momentum $q \to \lambda q$

$$\frac{1}{Q_{ij}^2} \to \frac{1}{2\,\lambda^2}\,\frac{1}{2\,p_i\,q}\,\max_{k\neq i,j}\left[\frac{p_i p_k}{(p_i+p_k)\,q} - \frac{m_i^2}{2\,p_i q}\right]$$

- (Quasi-)Collinear limit scale transverse momentum $k_{T,i} \to \lambda k_{T,i}$

$$\frac{1}{Q_{ij}^2} \to \frac{1}{2\,\lambda^2}\,\frac{1}{\left|p_{ij}^2 - m_i^2 - m_j^2\right|}\left(\tilde{C}_{i,j} + \tilde{C}_{j,i}\right)\,; \quad \tilde{C}_{i,j} = \begin{cases} \dfrac{z}{1-z} - \dfrac{m_i^2}{2\,p_i p_j} & \text{if } j = g \\ 1 & \text{else} \end{cases}$$

The ME⊗PS approach

How to interpret $\mathcal{P}^{(\mathrm{PS})\,\mathrm{PS}}_{\widetilde{ij},\tilde{k}}(t,t';\{\vec{a}\})$?

Assume predefined branchings at t and $t' > t$

$$\mathcal{K}_{ij,k}(t,z,\phi)\,\Theta\left[Q_{\mathrm{cut}} - Q_{ij,k}(t,z,\phi)\right]$$

means running a vetoed shower

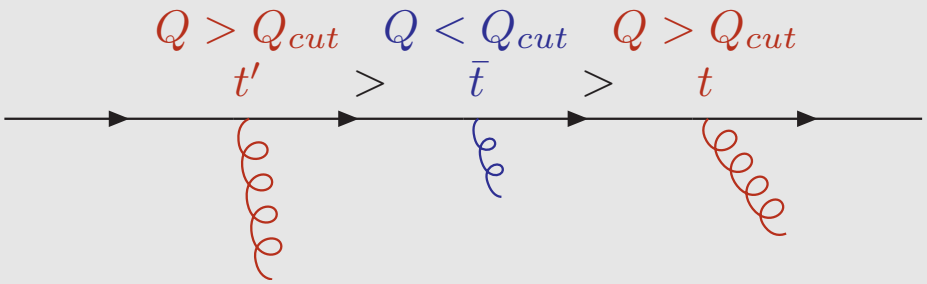
emission phase space is limited from above by Q_{cut}

$$\mathcal{P}^{(\mathrm{PS})\,\mathrm{PS}}_{\widetilde{ij},\tilde{k}}(t,t';\{\vec{a}\})$$

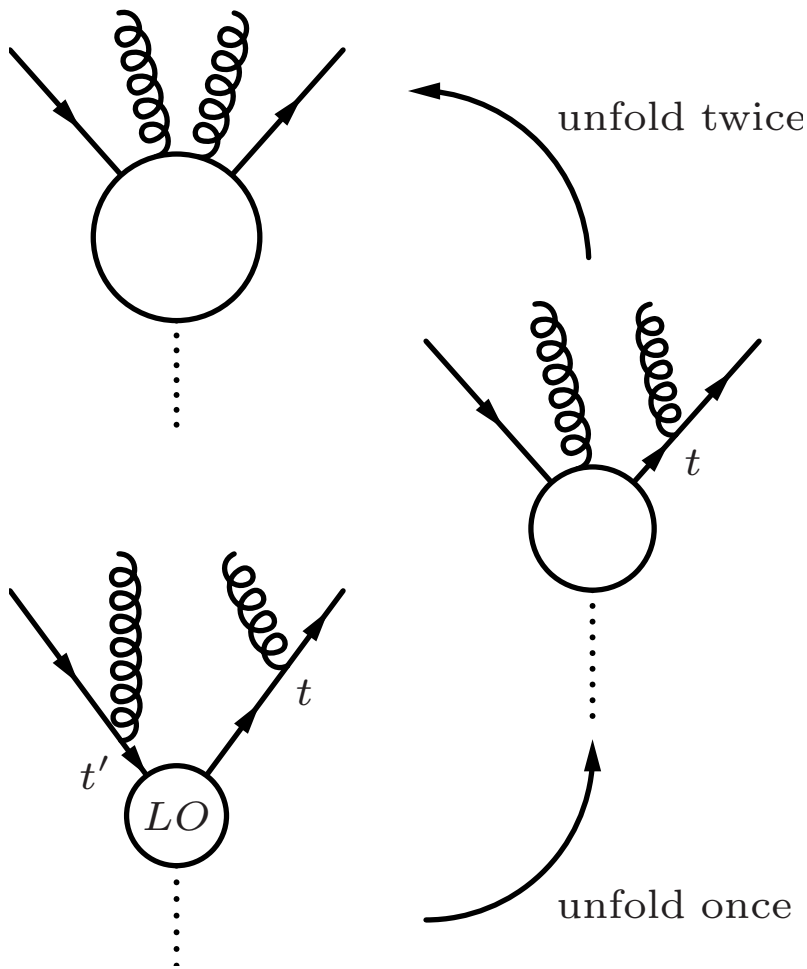
means running a truncated shower

t is larger than global shower cutoff t_0

What is the catch of it ?



Example branching history



The ME⊗PS approach

How to interpret $\mathcal{P}^{(\mathrm{PS})\,\mathrm{ME}}_{\widetilde{ij},\tilde{k}}(t,t';\{\vec{a}\})$?

Assume predefined branchings at t and $t' > t$

$$\mathcal{K}_{ij,k}(t,z,\phi)\,\Theta\left[Q_{ij,k}(t,z,\phi) - Q_{\mathrm{cut}}\right]$$

means running a vetoed shower

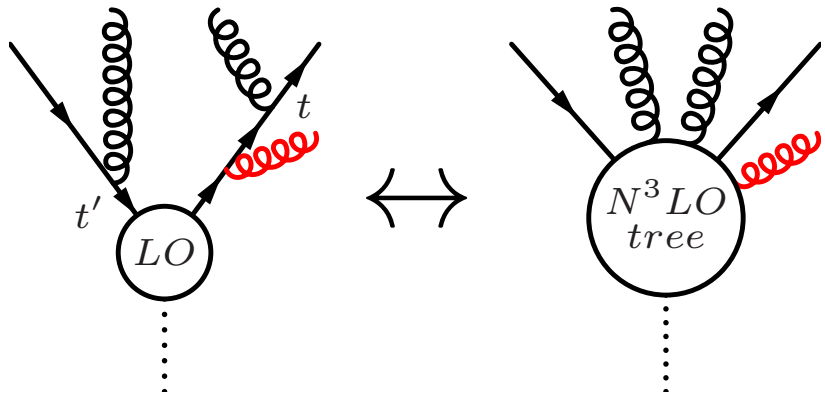
emission phase space is limited from below by Q_{cut}

$$\mathcal{P}^{(\mathrm{PS})\,\mathrm{ME}}_{\widetilde{ij},\tilde{k}}(t,t';\{\vec{a}\})$$

means running a truncated shower

t is larger than global shower cutoff t_0

Example emission



What happens if we emit something ?

Emission must be implemented to preserve full QCD evolution, i.e. $\mathcal{P}^{(\mathrm{PS})}_{\widetilde{ij},\tilde{k}}(t,t')$

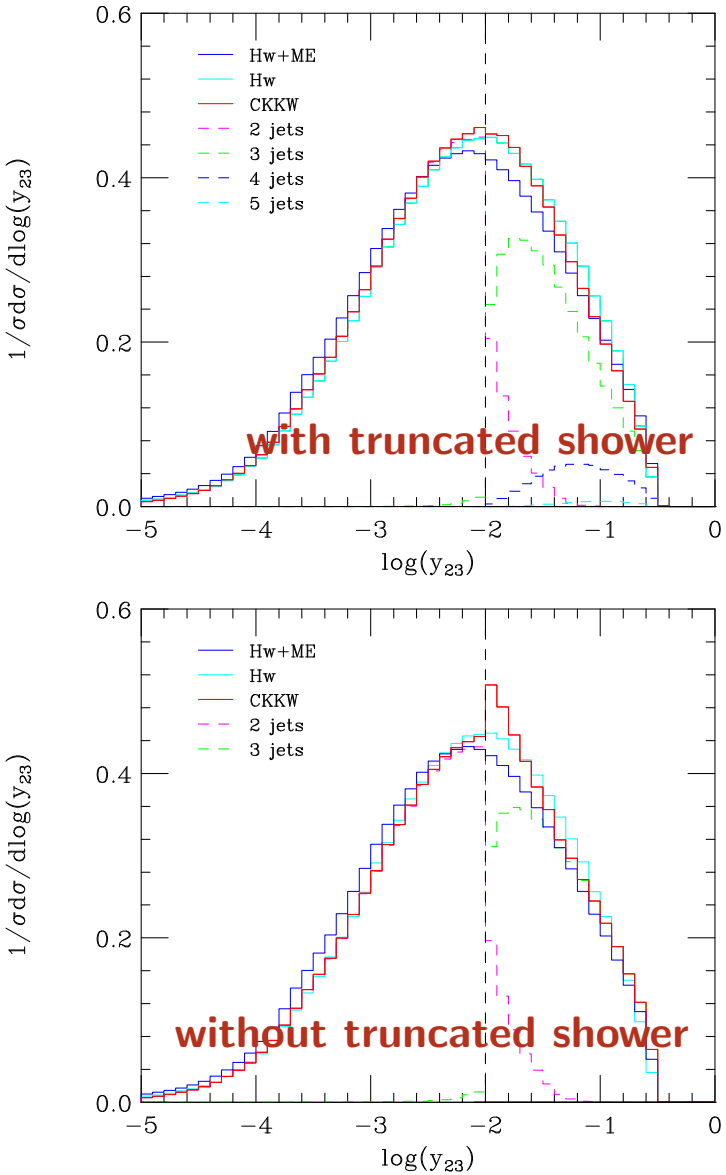
But we want matrix elements to take care of such emissions !

To avoid double-counting, the complete event must be rejected

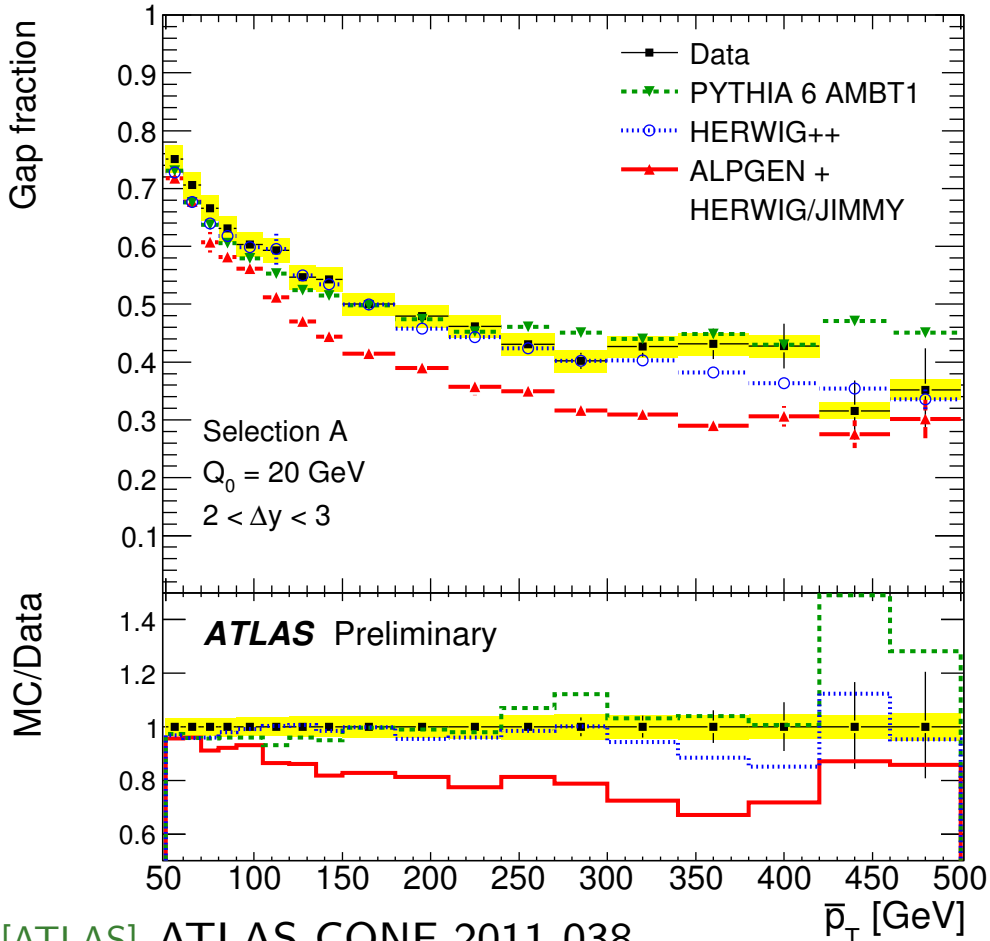
Event is lost ⇒ rejection reduces initial cross section σ to $\sigma \cdot \mathcal{P}^{(\mathrm{PS})\,\mathrm{ME}}_{\widetilde{ij},\tilde{k}}(t,t')$

"Gap" is filled by higher order ME⊗PS ⇒ σ preserved up to PS↔ME mismatch

The importance of truncated parton showers



[Hamilton,Richardson,Tully] JHEP11(2009)038



[ATLAS] ATLAS-CONF-2011-038

- Truncated PS necessary to maintain logarithmic accuracy if jet measure deviates from evolution variable
- Most prominent in fHERWIG/HERWIG++

Other tree-level merging schemes

Various other schemes for merging ME & PS exist, based on similar ideas

- CKKW-L scheme ARIADNE JHEP05(2002)046, JHEP07(2005)054
  phasespace separation via k_T-algorithm
  rejection via pseudo-shower
  Dipole shower including low-x effects $k_T =: Q \Rightarrow$ truncated PS not needed
- MLM scheme ALPGEN NPB632(2002)343, JHEP01(2007)013
  phasespace separation via cone-like algorithm
  rejection via "jet matching"
  Geometrical picture, no truncated shower would be needed!
- CKKW scheme SHERPA before v1.2.0 JHEP11(2001)063, JHEP08(2002)015
  phasespace separation via k_T-algorithm
  rejection via Sudakov weight from NLL formalism
  NLL Sudakov vs. PS rejection, no truncated shower would be needed!
- mixed schemes MADGRAPH, HELAC JHEP02(2009)017
  employing techniques similar to MLM / CKKW

Comparisons in DY-like processes mostly show reasonable agreement

hep-ph/0602031, EPJC53(2008)473 & arXiv:1003.1643 [hep-ph]

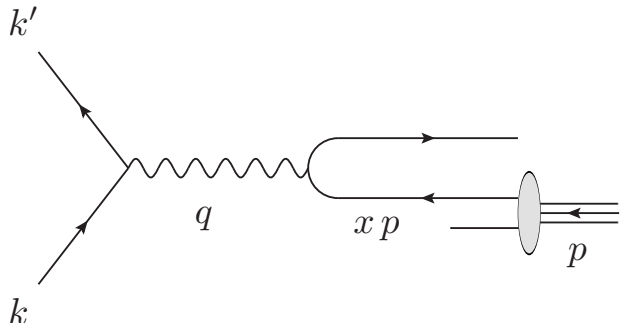
Inclusive ME⊗PS: What did we learn at HERA?

Leading order $e^{\pm}p$ - scattering in collinear factorization Breit frame

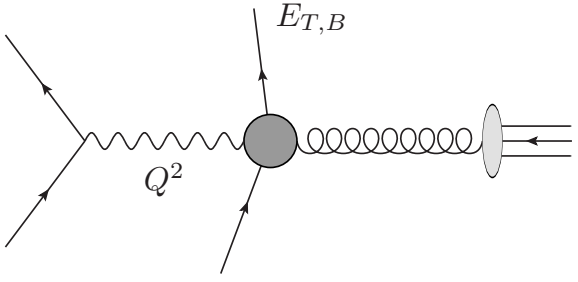
- **There are no jets !**
- Kinematical variables

$$Q^2 = q^2 = (k' - k)^2 \text{ and } x = \frac{Q^2}{2\, q \cdot p}$$

- Hadronic cm energy $W = Q\sqrt{(1-x)/x}$



What happens at higher orders ?



- **Multiple QCD scales, e.g. $E^2_{T,B}$**
- $e^{\pm}q \to e^{\pm}q$ if $E^2_{T,B} \lesssim Q^2$
  $\gamma^* g \to jets$ if $Q^2 \lesssim E^2_{T,B}$

What makes DIS any different from $e^+e^- \to jets$ and $pp \to e^+e^-$?

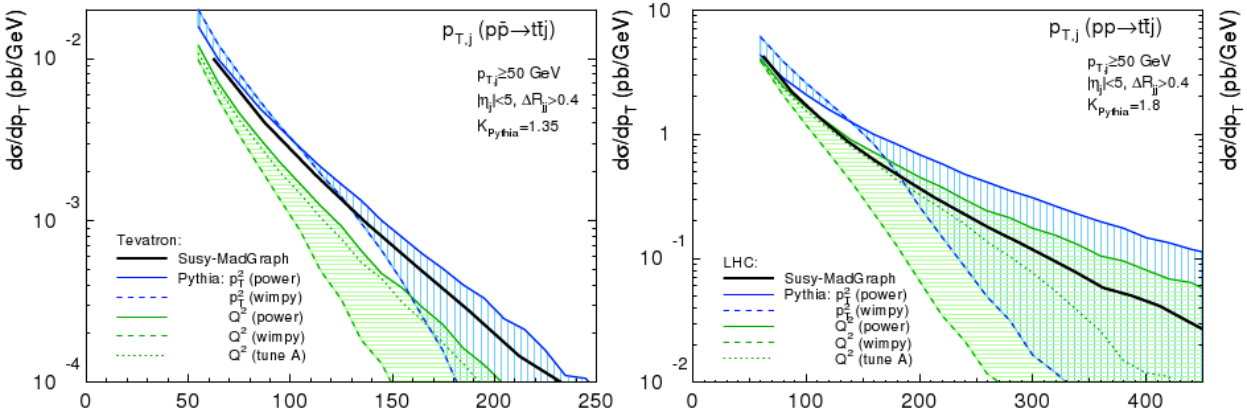
The virtuality of the exchanged photon tends to be close to zero !

N.B. This is also the case in the Drell-Yan process $pp \to e^+e^-$,
but recent experimental studies usually focus on $m_{l\bar{l}} \approx m_Z$

Inclusive ME⊗PS: What did we learn at HERA?

- Leading-order process does not define sensible scale for jets, but
- Factorization scale should be Q^2 to maintain total cross section
- Higher-order corrections usually large due to large phase space ($\propto W^2$)

Well-known problem in the context of Drell-Yan lepton-pair and heavy resonances production at hadron colliders
e.g. $t\bar{t} + jets$ hep-ph/0511306



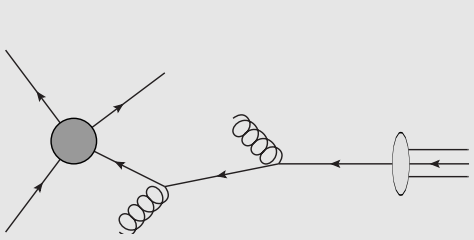
Often solved by using **"power shower"**
Increase starting scale of PS to open full phase space for emissions

Questionable concept for several reasons

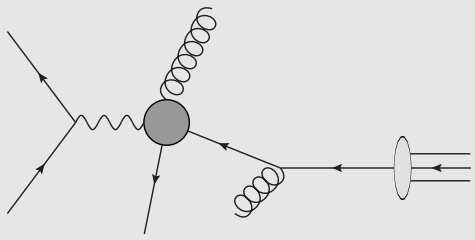
- Emissions might now be too hard arXiv:1003.2384 [hep-ph]
- Factorization theorems are not respected

Inclusive ME⊗PS: What did we learn at HERA?

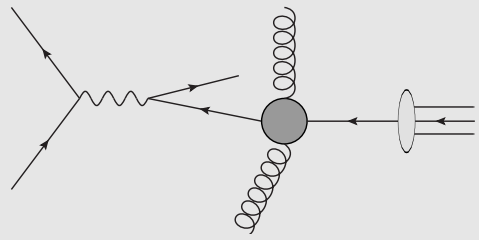
Different approach: Choose LO process dynamically, based on kinematics
Hard jets belong to "core" process rather than PS correction ⇒ PS must include QED & EW



$E^2_{T,B\,1} \approx E^2_{T,B\,2} < Q^2$



$Q^2 \approx E^2_{T,B\,2} < E^2_{T,B\,1}$



$Q^2 < E^2_{T,B\,1} \approx E^2_{T,B\,2}$

When does a jet become "hard" ?
Modify jet criterion to prevent QED/EW part from restricting QCD scales

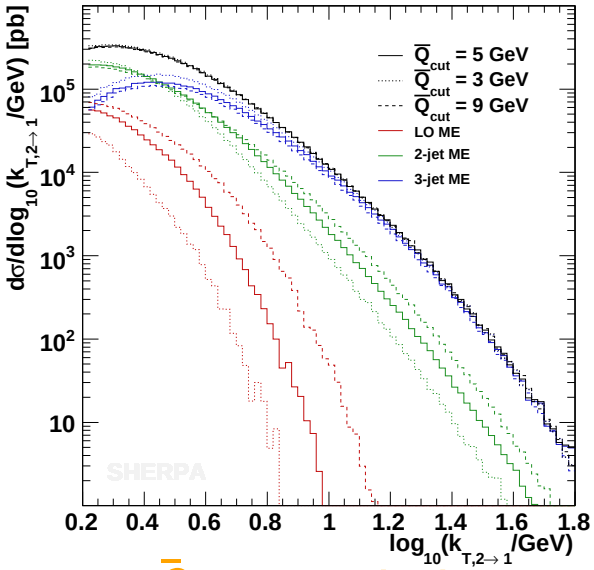
$$\frac{1}{Q^2_{\text{cut}}} = \frac{1}{\bar{Q}^2_{\text{cut}}} + \frac{1}{S^2_{\text{DIS}}\,Q^2}$$

Two essentially free parameters i.e. one more than in original ME⊗PS

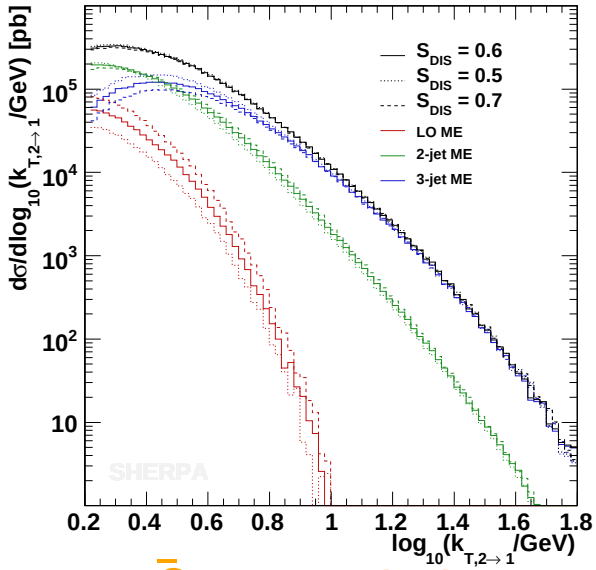
- $\bar{Q}_{\text{cut}}$ - classifies high-$E_{T,B}$ jets as hard ⇒ hard jets always from ME
- S_{DIS} - **classifies low-Q^2 photons as soft** ⇒ **jets from ME if $E^2_{T,B} > S^2_{\text{DIS}}\,Q^2$**

Reasonable range: $0.4 \lesssim S_{\text{DIS}} \lesssim 0.8$ ⇒ nonzero PS phase space at low Q^2

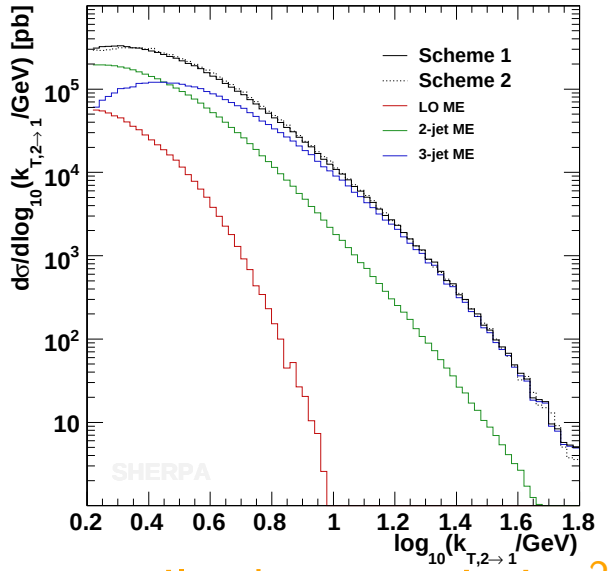
ME⊗PS in DIS: Stability tests



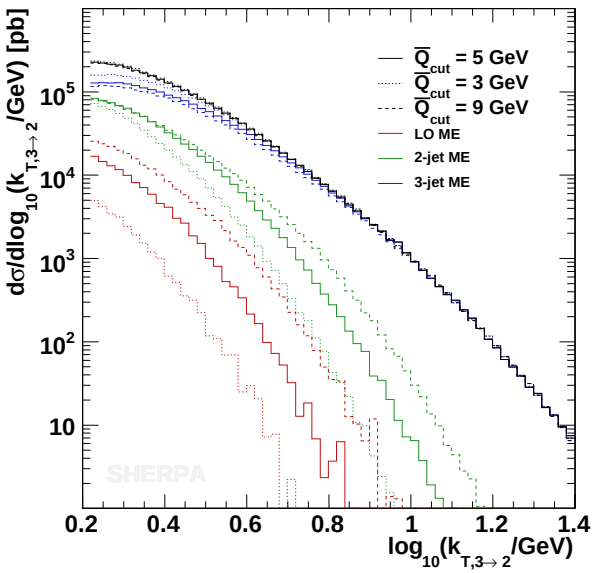
$\bar{Q}_{\mathrm{cut}}$ - variation

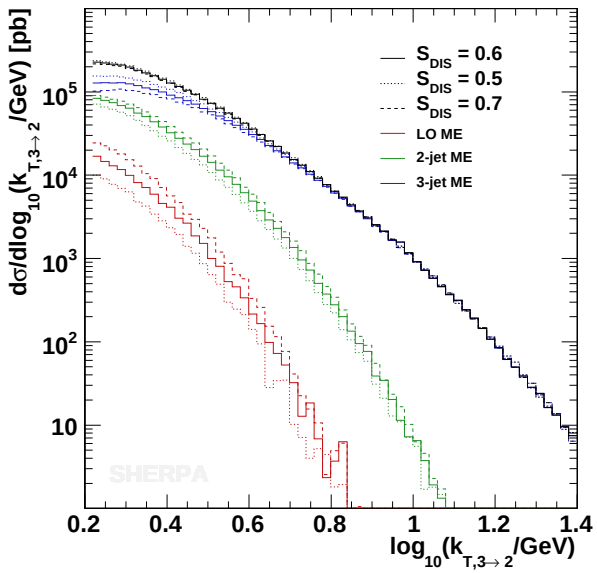


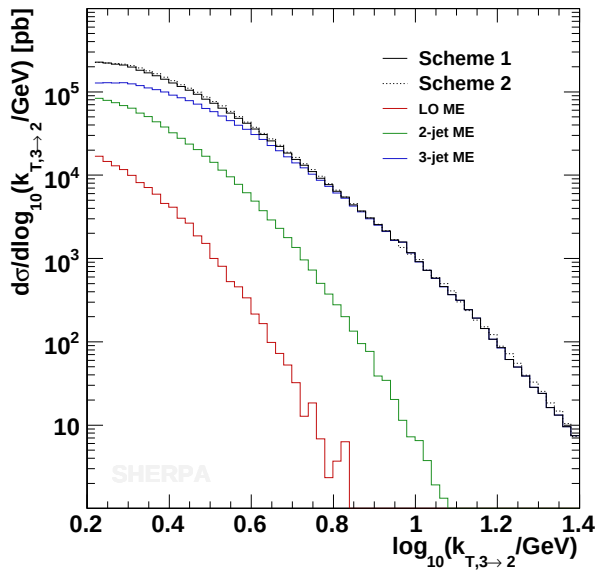
$\bar{S}_{\mathrm{DIS}}$ - variation



recoil scheme variation[2]



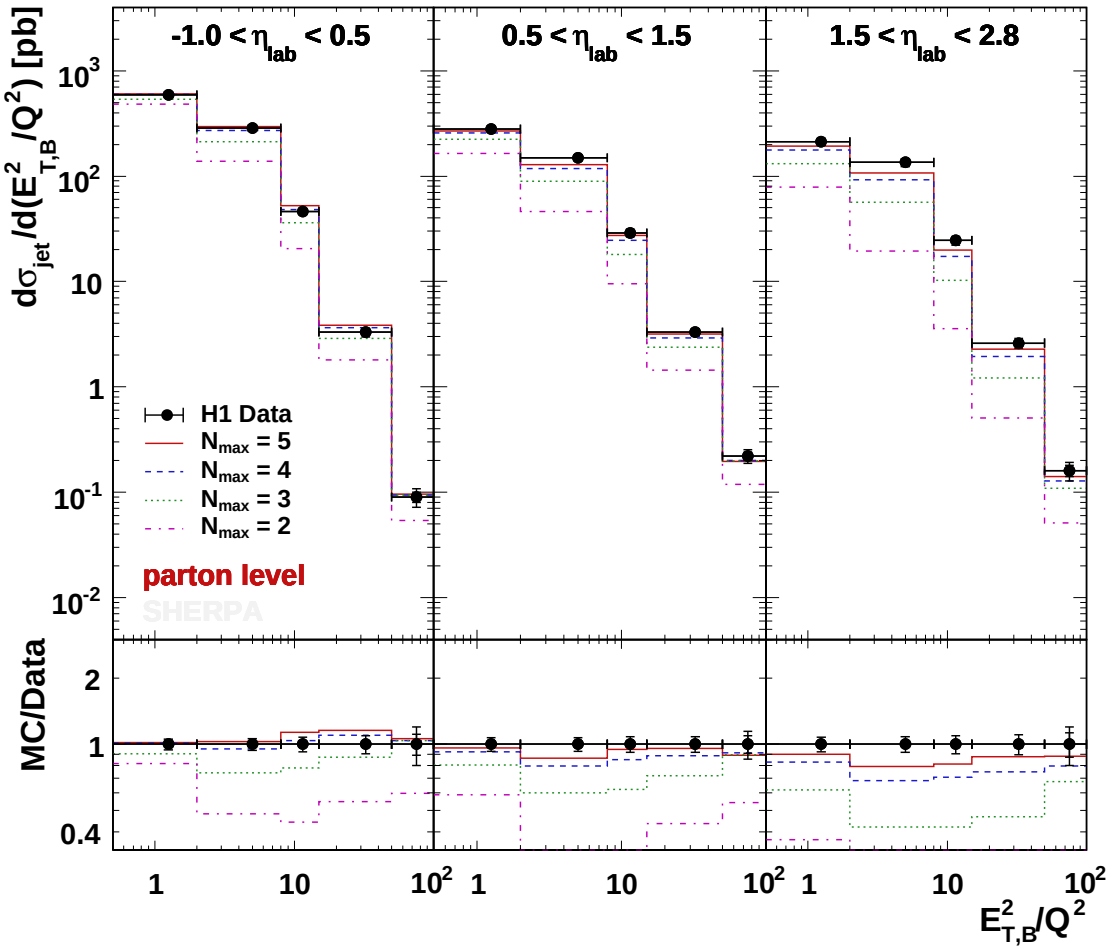


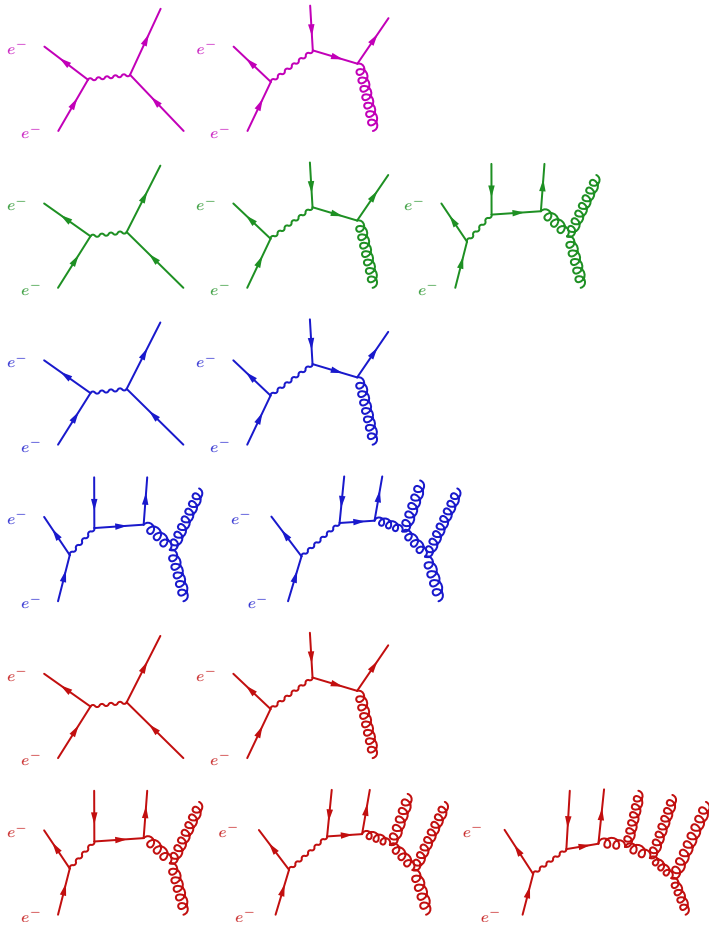


[2] see PRD81(2010)034026

ME⊗PS results: Inclusive jets in DIS PLB542(2002)193

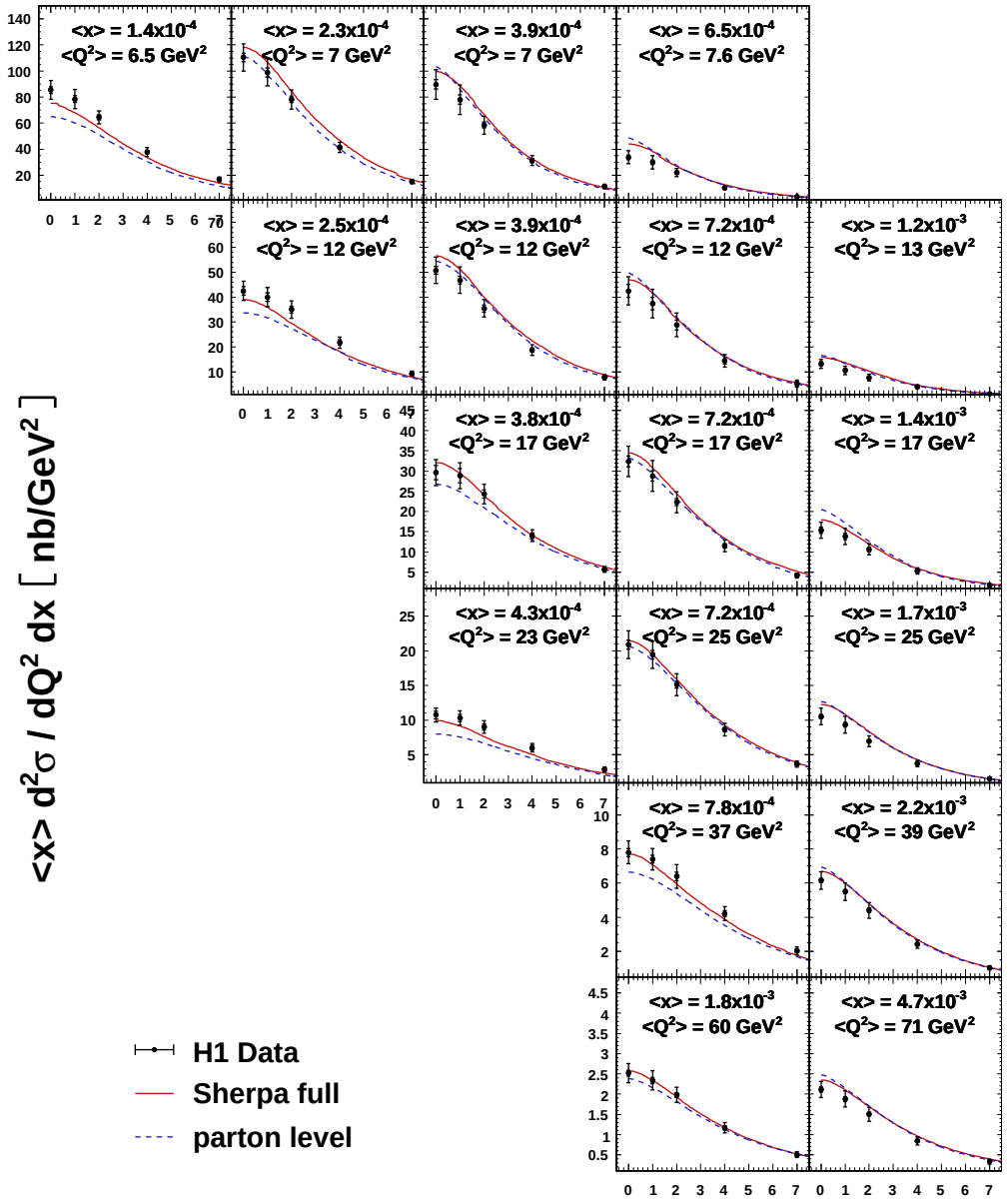
Variation of maximum matrix-element multiplicity, $N_{\max}$ [1]





[1] SHERPA ⊕ HZTool from EPJC67(2010)73

ME⊗PS results: Low-x di-jets in DIS EPJC33(2004)477

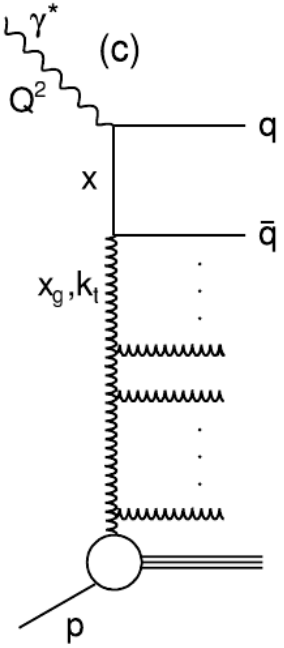


Δ in bins of $\langle x \rangle$ and $\langle Q^2 \rangle$

Δ defined as $E^*_{T,\max} > E^*_{T\,\mathrm{cut}} + \Delta$

$E^*_{T\,\mathrm{cut}} \rightarrow$ minimum jet transverse energy

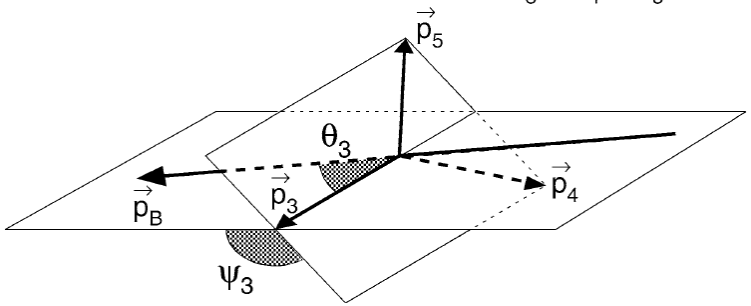
$E^*_{T\,\max} \rightarrow$ transverse energy of hardest jet



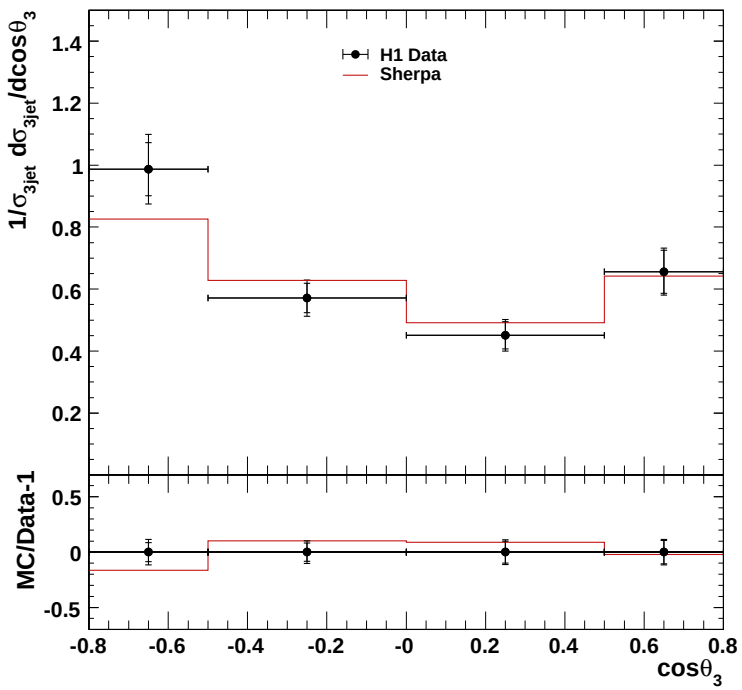
ME⊗PS results: Inclusive tri-jets in DIS PLB515(2001)17

three-jet center-of-mass frame:

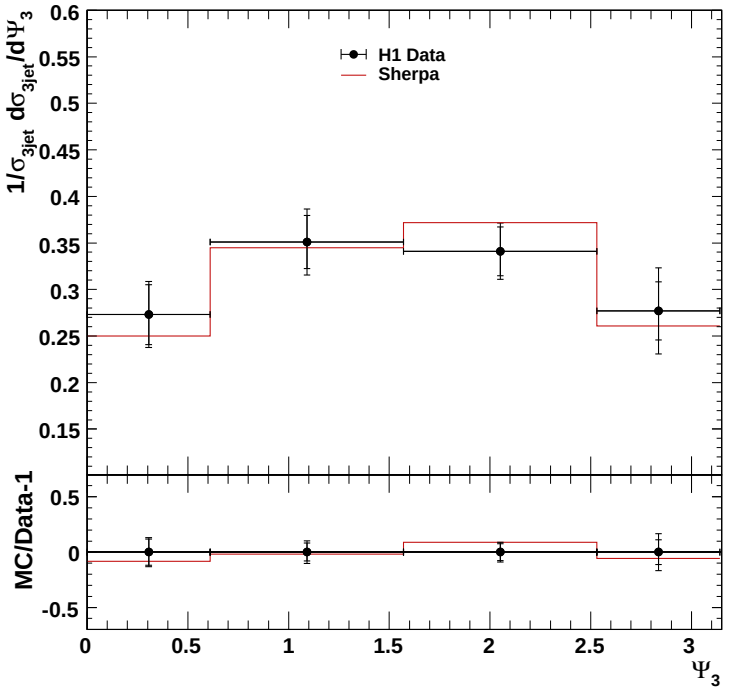
$1 + 2 \rightarrow 3 + 4 + 5$ $\quad E_3 > E_4 > E_5$



$\cos\theta_3$ $Q^2 > 150$ GeV^2



Ψ_3 $Q^2 > 150$ GeV^2



Why investigate photon production?

Jet definition at low E_T

- "Clean" $\gamma + jet$ signature defines jet characteristics via p_T balance
- Detector response to γ well known

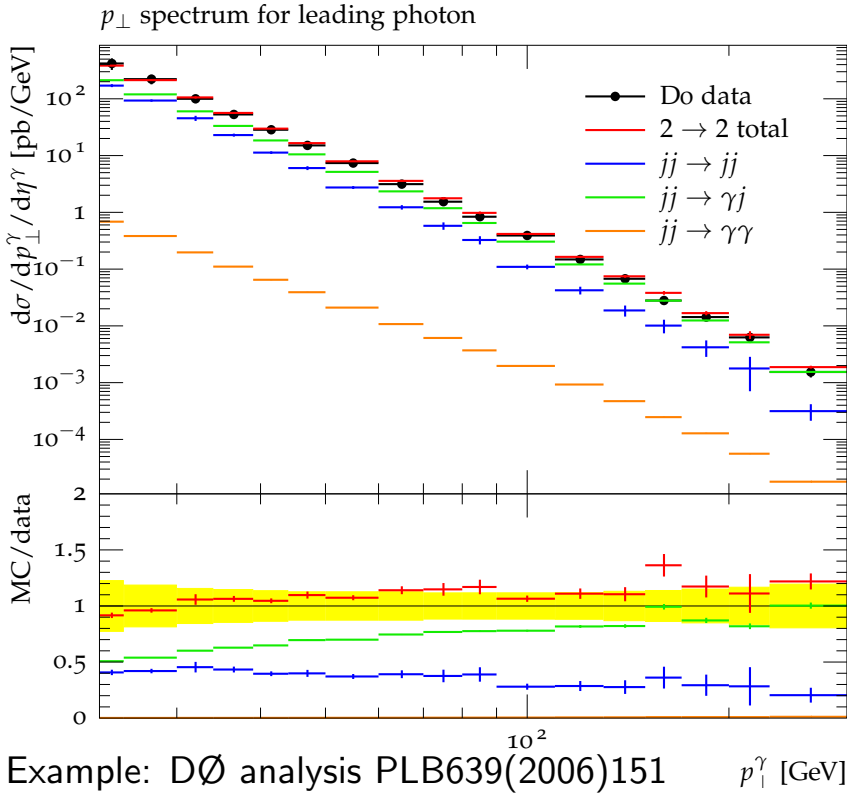
New & old physics background

- $h \to \gamma\gamma$ (+ jets)
- BSM signatures often include γ's
- High-p_T γ's interesting for AGC's

Similarity to QCD

- Resummation possible YFS
- Often need to improve with NLO
- High-p_T tails often interesting

MC simulation often treacherous



Example: DØ analysis PLB639(2006)151

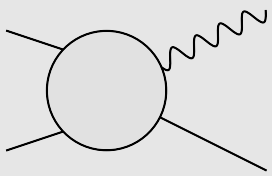
despite strict cuts on photon p_T $p_T > 23\,\mathrm{GeV}$ and isolation requirements $E_{\mathrm{EM}}/E_{\mathrm{tot}} > 0.95$

Large fraction of events from $jj \to jj$!

The perturbative QCD approach

"Direct" component

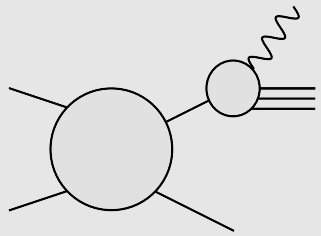
fixed-order calculation with isolated γ



- γ+jet (JetPhox) PRD73(2006)094007
- $\gamma\gamma$ (DiPhox) EPJC16(2000)311
- $\gamma\gamma$+jet JHEP04(2003)059
- $gg \to \gamma\gamma g$ PLB460(1999)184

"Fragmentation" component

factorization of $q\gamma$ collinear singularity



- Gives rise to fragmentation function $D^{\gamma}_{q,g}$ PLB79(1978)83
- Contribution depends on photon isolation criterion

Prerequisites

- Photon isolation criterion for IR-safe cross-section definition
  cone PRD42(1990)61, smooth isolation PLB429(1998)369, democratic approach ZPC62(1994)311
- Nonperturbative input for $D^{\gamma}_{q,g}$ from measurements or model

Non-prompt component not included in this approach, but experimentally separable
$\pi^0 \to \gamma\gamma$, $\eta \to \gamma\gamma$, ...

The Monte Carlo approach

"Democratic" Monte Carlo model idea based on ZPC62(1994)311

- Treat partons and photons fully democratically
- Combine matrix elements of various parton/photon multiplicity with
- Interleaved QCD⊕QED parton shower evolution ⇒ evolution of $D^{\gamma}_{q,g}$

Modifying the parton shower

Add splitting functions for $qq\gamma$ similarly implemented in Ariadne, Herwig & Pythia

$$\mathcal{P}^{(\mathrm{PS})}(t,t') = \mathcal{P}^{(\mathrm{PS})\,\mathrm{QCD}}(t,t')\,\mathcal{P}^{(\mathrm{PS})\,\mathrm{QED}}(t,t')$$

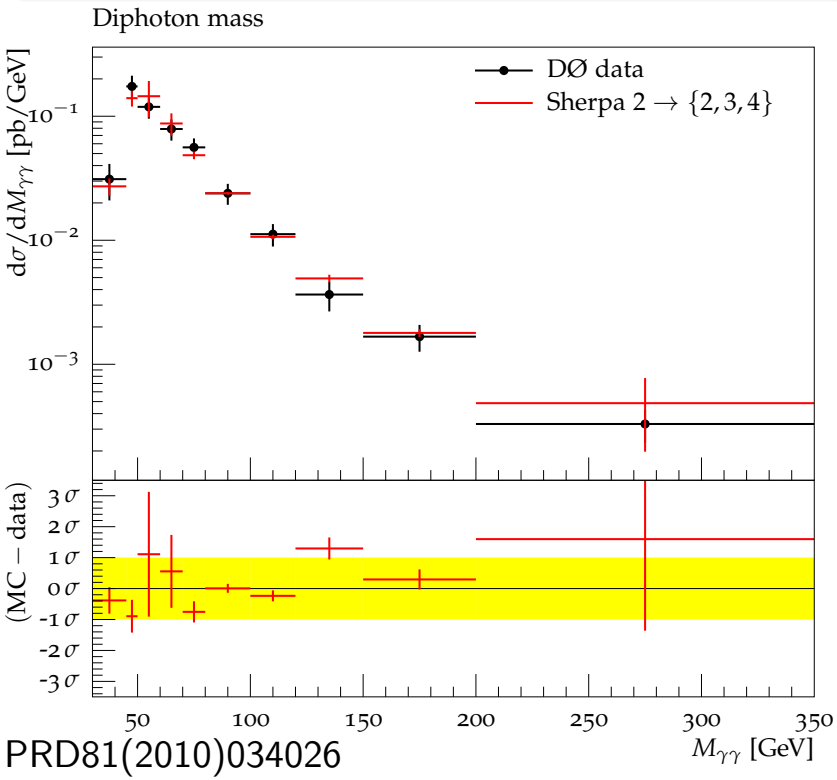
Here: Splitting kernels based on spin-averaged CS dipole functions
⇒ Recoil partners are all oppositely charged particles in the event
Spin dependence restored by real emission ME used to correct the PS

How to treat the nonperturbative regime ?

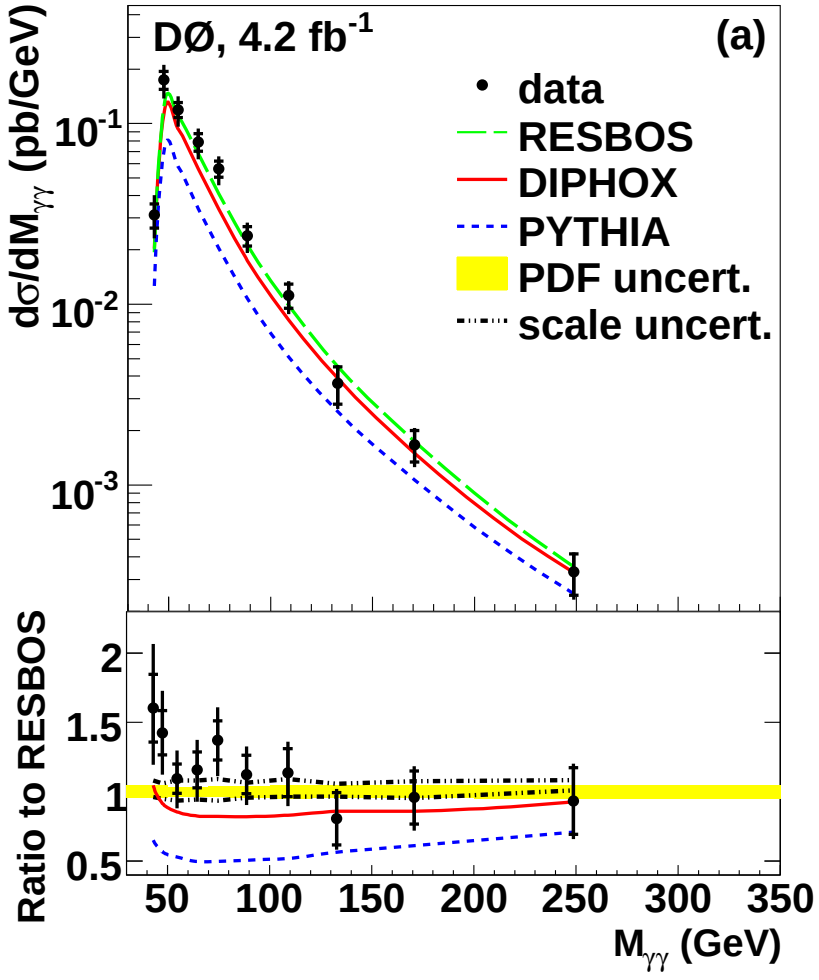
- Parton-to-hadron transition via fragmentation models cluster/string model
- Non-prompt γ-production for free via hadron decay packages
- No new parameters !

Diphoton production at the Tevatron PLB690(2010)108

$E_T^{\gamma 1} > 21$ GeV, $E_T^{\gamma 1} > 20$ GeV,
$|\eta^{\gamma}| < 0.9$, $E_T^{R=0.4} - E_T^{\gamma} < 2.5$ GeV



PRD81(2010)034026



SHERPA prediction: Merged $2 \to \{2+3+4\}$-jet/γ plus $gg \to \gamma\gamma box$

Diphoton production at the Tevatron PLB690(2010)108

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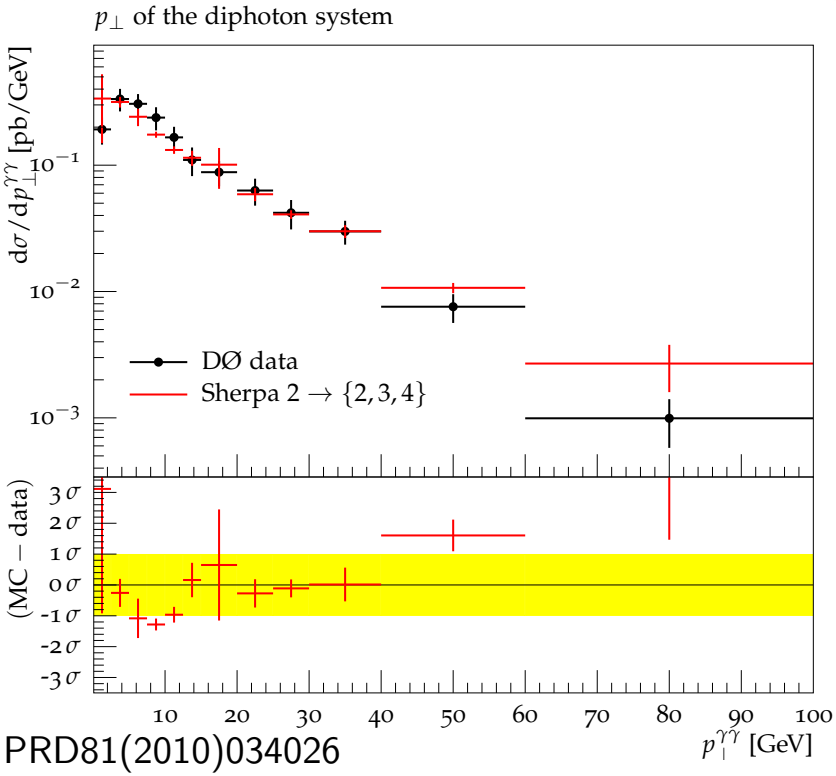
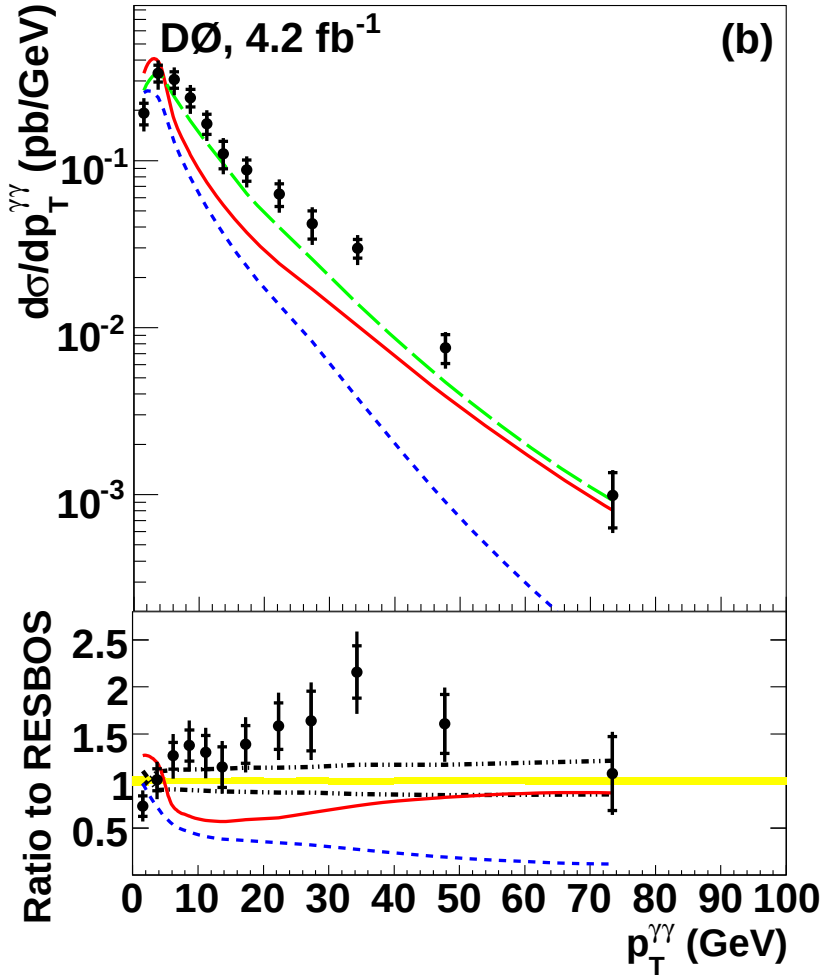
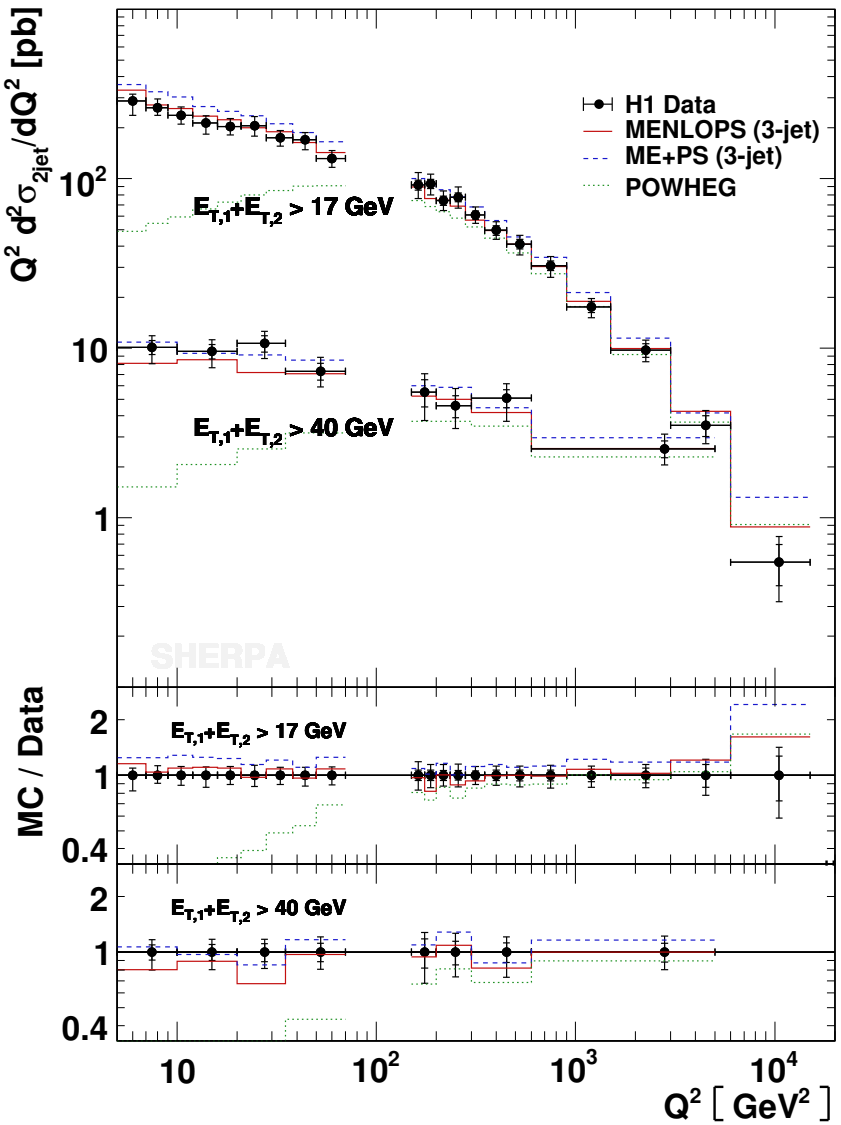


PRD81(2010)034026



SHERPA prediction: Merged 2→{2+3+4}-jet/γ plus $gg \to \gamma\gamma box$

A word of warning on MENLOPS



Data: [H1] EPJC19(2001)289

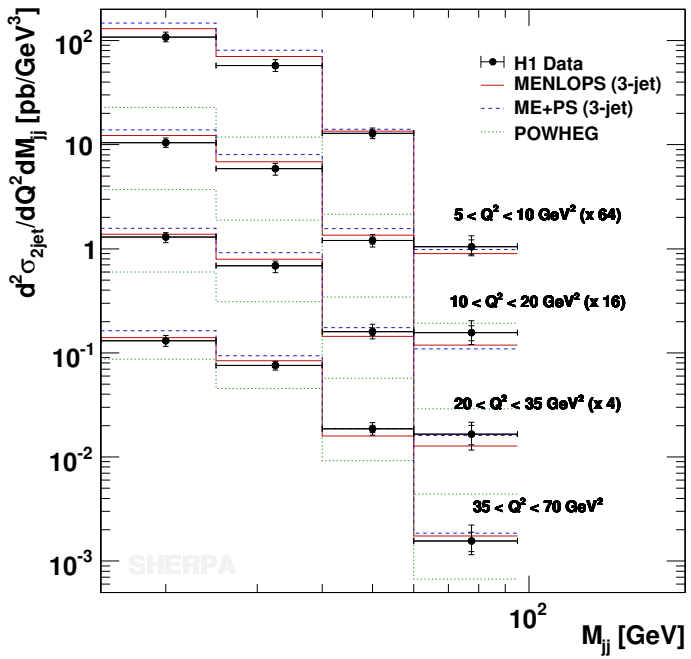
Can we improve DIS predictions ?

No way !

$Q^2 < \bar{E}_{T,B}$ is unlikely in POWHEG

→ same scale uncertainty from MENLOPS

Need *inclusive* MENLOPS to improve

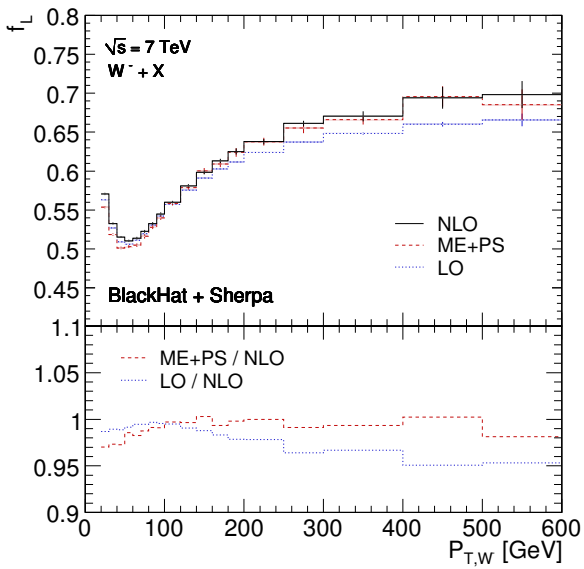


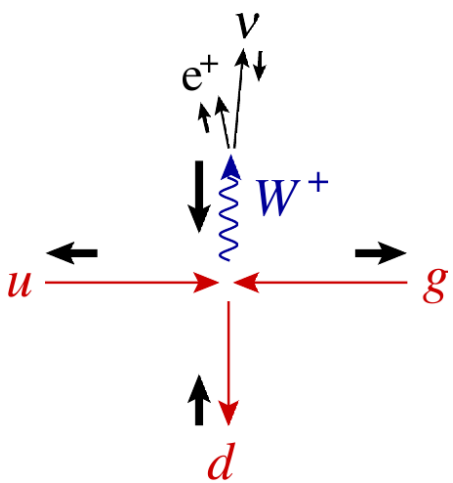
NLO at work: The left-handed W

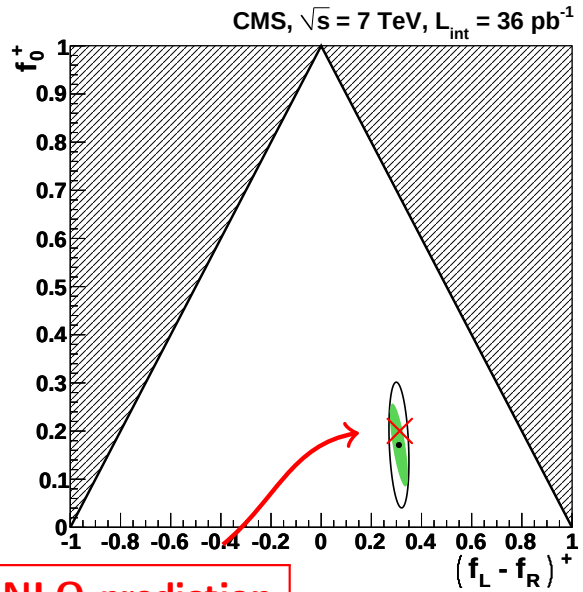
LO theory: W-bosons dominantly left-handed at large transverse momenta [BlackHat] arXiv:0912.4927
Useful to distinguish W+jets from $t\bar{t}$+jets & BSM processes

Effect persists at NLO
[BlackHat] PRD80(2009)074036 PRD84(2011)034008

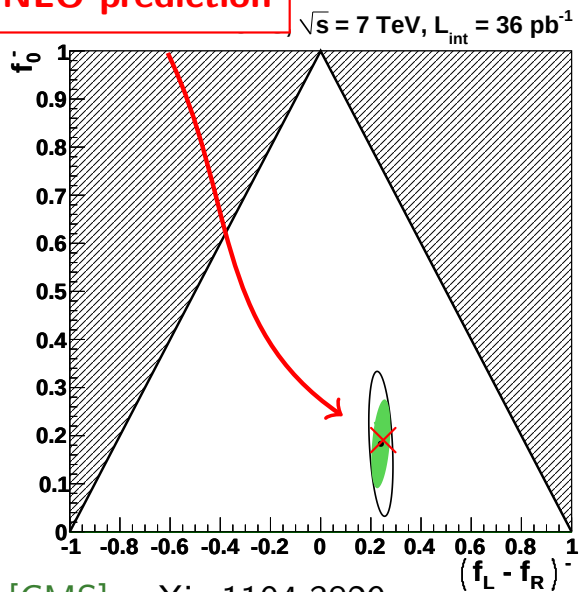
Uncertainty $\leq 10\%$, assessed by comparison with tree-level ME+PS merging approach







NLO prediction



[CMS] arXiv:1104.3829