Triple collinear splittings in parton showers

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Which problem are we trying to solve?

- NLO DGLAP splitting kernels known since long
  [Curci,Furmanski,Petronzio] NPB175(1980)27, PLB97(1980)437
  [Floratos,Kounnas,Lacaze] NPB192(1981)417
- So far not implemented in parton showers because
  - Kernels are scheme dependent (easy)
  - Overlap with soft-gluon resummation (hard)
- Focus on collinear branchings for a start
  - Redefine time-like Sudakovs to recover NLO DGLAP evolution
    [Jadach,Skrzypek] hep-ph/0312355
  - Negative NLO corrections require weighted veto algorithm
    [Schumann,Siegert,SH] arXiv:0912.3501, [Lönnblad] arXiv:1211.7204
  - Flavor changing splitting functions require $2 \to 4$ transitions
    [Prestel,SH] arXiv:1705.00742
- Flavor-changing case is simplest but requires all the technology

Parton-shower interpretation of the DGLAP equations

- DGLAP equation for fragmentation functions

$$\frac{\mathrm{d}x\,D_a(x,t)}{\mathrm{d}\ln t}=\sum_{b=q,g}\int_0^1\mathrm{d}\tau\int_0^1\mathrm{d}z\,\frac{\alpha_s}{2\pi}\big[zP_{ab}(z)\big]_+\,\tau D_b(\tau,t)\,\delta(x-\tau z)$$

- Define plus prescription $\big[zP_{ab}(z)\big]_+=\lim_{\varepsilon\to0}zP_{ab}(z,\varepsilon)$

$$P_{ab}(z,\varepsilon)=P_{ab}(z)\,\Theta(1-z-\varepsilon)-\delta_{ab}\sum_{c\in\{q,g\}}\frac{\Theta(z-1+\varepsilon)}{\varepsilon}\int_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,P_{ac}(\zeta)$$

- Rewrite for finite ε

$$\frac{\mathrm{d}\ln D_a(x,t)}{\mathrm{d}\ln t}=-\sum_{c=q,g}\int_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,\frac{\alpha_s}{2\pi}P_{ac}(\zeta)+\sum_{b=q,g}\int_x^{1-\varepsilon}\frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)\,\frac{D_b(x/z,t)}{D_a(x,t)}$$

- First term is derivative of Sudakov factor

$$\Delta_a(t_0,t)=\exp\bigg\{-\int_{t_0}^{t}\frac{\mathrm{d}\bar{t}}{\bar{t}}\sum_{c=q,g}\int_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,\frac{\alpha_s}{2\pi}P_{ac}(\zeta)\bigg\}$$

Parton-shower interpretation of the DGLAP equations

- Use generating function $\mathcal{D}_a(x,t,\mu^2) = D_a(x,t)\Delta_a(t,\mu^2)$ to write

$$\frac{\mathrm{d}\ln\mathcal{D}_a(x,t,\mu^2)}{\mathrm{d}\ln t} = \sum_{b=q,g}\int_x^{1-\varepsilon}\frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)\,\frac{D_b(x/z,t)}{D_a(x,t)}\;.$$

- Generalize to hadronic collision with PDFs & fragmenting jet functions

$$\frac{\mathrm{d}\ln\mathcal{F}_{\vec{a}}(\vec{x},t,\mu^2)}{\mathrm{d}\ln t} = \sum_{i\in\mathrm{IS}}\sum_{b=q,g}\int_{x_i}^{1-\varepsilon}\frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi}\,P_{ba_i}(z)\,\frac{f_b(x_i/z,t)}{f_{a_i}(x_i,t)} + \sum_{j\in\mathrm{FS}}\sum_{b=q,g}\int_{x_j}^{1-\varepsilon}\frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi}\,P_{a_jb}(z)\,\frac{\mathcal{G}_b(x_j/z,t)}{\mathcal{G}_{a_j}(x_j,t)}\;.$$

- If jets not measured, obtain parton-shower generating function

$$\frac{\mathrm{d}}{\mathrm{d}\ln t}\ln\left(\frac{\mathcal{F}_{\vec{a}}(\vec{x},t,\mu^2)}{\prod_{j\in\mathrm{FS}}\mathcal{G}_{a_j}(x_j,t)}\right) = \sum_{i\in\mathrm{IS}}\sum_{b=q,g}\int_{x_i}^{1-\varepsilon}\frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi}\,P_{ba_i}(z)\,\frac{f_b(x_i/z,t)}{f_{a_i}(x_i,t)} + \sum_{j\in\mathrm{FS}}\sum_{b=q,g}\int_0^{1-\varepsilon}\mathrm{d}z\,z\,\frac{\alpha_s}{2\pi}\,P_{a_jb}(z)\;.$$

Symmetry factors at LO and at NLO

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- Net result: Unitarity implies that forward Sudakovs computed with symmetry factors replaced by z [Jadach,Skrzypek] hep-ph/0312355
- Convenient interpretation as “tagging” of evolving parton
- Equivalent to standard technique at LO, because

$$\sum_{b=q,g}\int_0^{1-\varepsilon}\mathrm{d}z\,z\,P_{qb}^{(0)}(z)=\int_\varepsilon^{1-\varepsilon}\mathrm{d}z\,P_{qq}^{(0)}(z)+\mathcal{O}(\varepsilon)\;,$$

$$\sum_{b=q,g}\int_0^{1-\varepsilon}\mathrm{d}z\,z\,P_{gb}^{(0)}(z)=\int_\varepsilon^{1-\varepsilon}\mathrm{d}z\left[\,\frac{1}{2}P_{gg}^{(0)}(z)+n_f\,P_{gq}^{(0)}(z)\,\right]+\mathcal{O}(\varepsilon)\;.$$

- More care is needed at NLO

$$\sum_{\substack{b=q,g\\b\neq a}}\int_0^{1-\varepsilon}\mathrm{d}z_1\int_0^{1-\varepsilon}\mathrm{d}z_2\,\frac{z_1\,z_2}{1-z_1}\,\Theta(1-z_1-z_2)$$
$$\times\Big(P_{a\to ba\bar{b}}(z_1,z_2,\ldots)+P_{a\to b\bar{b}a}(z_1,z_2,\ldots)\Big)$$
$$=\sum_{\substack{b=q,g\\b\neq a}}\int_\varepsilon^{1-\varepsilon}\mathrm{d}z_1\int_\varepsilon^{1-z_1}\mathrm{d}z_2\,\frac{1}{\prod_{i=q,g}n_i!}\,P_{a\to ba\bar{b}}(z_1,z_2,\ldots)+\mathcal{O}(\varepsilon)$$

$2 \to 4$ kinematics mapping, FF case

- Define evolution & splitting variables

$$t = \frac{4\, p_j p_{ai}\, p_{ai} p_k}{q^2 - m_{aij}^2 - m_k^2}\;, \qquad z_a = \frac{2\, p_a p_k}{q^2 - m_{aij}^2 - m_k^2}$$

$$s_{ai} = 2\, p_a p_i + m_a^2 + m_i^2\;, \qquad x_a = \frac{p_a p_k}{p_{ai} p_k}$$

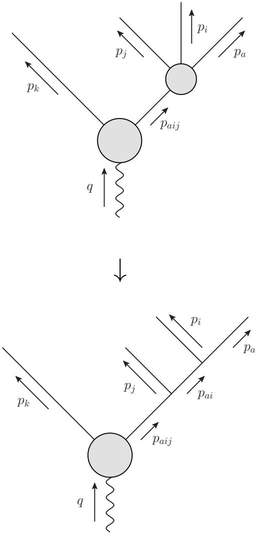
- First branching $(\widetilde{aij}, \tilde{k}) \to (ai, j, k)$ constructed with $m_{ai}^2 \to s_{ai}$, using
  [Catani,Dittmaier,Seymour,Trocsanyi] hep-ph/0201036
  [Prestel,SH] arXiv:1506.05057

$$y = \frac{t\, x_a / z_a}{q^2 - s_{ai} - m_j^2 - m_k^2}$$

$$\tilde{z} = \frac{z_a / x_a}{1 - y}\, \frac{q^2 - m_{aij}^2 - m_k^2}{q^2 - s_{ai} - m_j^2 - m_k^2}\;.$$

- Second step now a decay $(ai, k) \to (a, i, k)$ can use CDST algorithm with

$$y' = \left[1 + \frac{z_a}{x_a}\, \frac{q^2 - m_{aij}^2 - m_k^2}{s_{ai} - m_a^2 - m_i^2}\right]^{-1}\;, \qquad \tilde{z}' = x_a$$



$2 \to 4$ phase space, FF case

- Phase space factorization derived similar to [Dittmaier] hep-ph/9904440
$\to$ s-channel factorization over p_{aij}, subsequently over p_{ai}

$$\int \mathrm{d}\Phi(p_a, p_i, p_j, p_k | q) = \int \frac{\mathrm{d}s_{aij}}{2\pi} \int \mathrm{d}\Phi(p_{aij}, p_k | q) \int \mathrm{d}\Phi(p_a, p_i, p_j | p_{aij})$$
$$= \int \mathrm{d}\Phi(\tilde{p}_{aij}, \tilde{p}_k | q) \int \Big[\mathrm{d}\Phi(p_a, p_i, p_j | \tilde{p}_{aij}, \tilde{p}_k)\Big]$$

- Obtain almost fully factorized form

$$\int \Big[\mathrm{d}\Phi(p_a, p_i, p_j | \tilde{p}_{aij}, \tilde{p}_k)\Big] =$$
$$\left[\frac{J^{(1)}_{\mathrm{FF}}}{4(2\pi)^3} \int \frac{\mathrm{d}t}{t} \int \mathrm{d}z_a \int \mathrm{d}\phi_j\right]\left[\frac{1}{4(2\pi)^3} \int \mathrm{d}s_{ai} \int \frac{\mathrm{d}x_a}{x_a} \int \mathrm{d}\phi_i\, J^{(2)}_{\mathrm{FF}}\right] 2\, p_{ai} p_j$$

- Simple Jacobians that reduce to unity for massless partons

$$J^{(1)}_{\mathrm{FF}} = \frac{q^2 - m^2_{aij} - m^2_k}{\sqrt{\lambda(q^2, m^2_{aij}, m^2_k)}}\,, \qquad J^{(2)}_{\mathrm{FF}} = \frac{s_{aik} - s_{ai} - m^2_k}{\sqrt{\lambda(s_{aik}, s_{ai}, m^2_k)}}\,.$$

Collinear factorization and resummation

- Combination with massless matrix element in collinear limit leads to

$$\int \mathrm{d}\Phi(p_a,p_i,p_j,p_k|\,q)\,|M_{n+2}(a,i,j,k|\,q)|^2$$
$$= \int\frac{\mathrm{d}t}{t}\int\mathrm{d}z_a\int\mathrm{d}s_{ai}\int\frac{\mathrm{d}x_a}{x_a}\int\frac{\mathrm{d}\phi_i}{2\pi}\,\frac{2p_{ai}p_j}{s_{aij}}$$
$$\times\left(\frac{\alpha_s}{2\pi}\right)^2\frac{P_{(aij)a}(p_a,p_i,p_j)}{s_{aij}}\int\mathrm{d}\Phi(\tilde{p}_{aij},\tilde{p}_k|\,q)\,|M_n(\widetilde{aij},\tilde{k}|\,q)|^2$$

- Write as differential branching probability

$$\frac{\mathrm{d}\ln\Delta^{1\to3}_{(aij)a}}{\mathrm{d}\ln t}=\int\mathrm{d}z_a\,\frac{z_a z_i}{1-z_a}\int\mathrm{d}s_{ai}\int\frac{\mathrm{d}x_a}{x_a}\int\frac{\mathrm{d}\phi_i}{2\pi}\,\left(\frac{\alpha_s}{2\pi}\right)^2\frac{P_{(aij)a}(p_a,p_i,p_j)}{s_{aij}^2/\,2\,p_{ai}p_j}$$

- LO PS accounts for iterated collinear limit, hence we must subtract

$$\frac{\mathrm{d}\ln\Delta^{(1\to2)^2}_{(aij)a}}{\mathrm{d}\ln t}=\int\mathrm{d}z_a\,\frac{z_a z_i}{1-z_a}\int\frac{\mathrm{d}s_{ai}}{s_{ai}}\int\frac{\mathrm{d}\xi}{\xi}\,\left(\frac{\alpha_s}{2\pi}\right)^2\frac{\sum_{(ai)}P^{(0)}_{(aij)(ai)}(\xi)P^{(0)}_{(ai)a}(z_a/\xi)}{s_{aij}/\,2\,p_{ai}p_j}$$

Collinear factorization and resummation

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- Simplest possible configuration $q \to q'$ [Catani,Grazzini] hep-ph/9908523

$$P_{qq'} = \frac{1}{2} C_F T_R \frac{s_{aij}}{s_{ai}} \left[-\frac{t^2_{ai,j}}{s_{ai} s_{aij}} + \frac{4\, z_j + (z_a - z_i)^2}{z_a + z_i} + (1 - 2\varepsilon) \left(z_a + z_i - \frac{s_{ai}}{s_{aij}} \right) \right]$$

where $(z_a + z_i)\, t_{ai,j} = 2(z_a s_{ij} - z_i s_{aj}) + (z_a - z_i) s_{ai}$

- Apparent collinear singularity in s_{ai} that cancels upon azimuthal averaging against iterated LO splitting
- But integrand locally divergent $\to$ not amenable to MC simulation
- Solved by subtraction of spin-correlated LO splitting functions [Somogyi,Trocsanyi,del Duca] hep-ph/0502226

$$P^{\mu\nu}_{qg} = C_F \left[-2 \frac{z}{1-z} \frac{k_T^\mu k_T^\nu}{k_T^2} + \frac{1-z}{2} \left(-g^{\mu\nu} + \frac{p^\mu n^\nu + p^\nu n^\mu}{np} \right) \right]$$

$$P^{\mu\nu}_{gq} = T_R \left[-g^{\mu\nu} + 4\, z(1-z)\, \frac{k_T^\mu k_T^\nu}{k_T^2} \right]$$

- Leads to additional subtraction term

$$\Delta P_{qq'} = C_F T_R \frac{4 z_a z_i z_j}{(1-z_j)^3} \left(1 - 2\cos^2\phi \right) , \qquad \cos\phi = \frac{s_{ai} s_{jk} + s_{ak} s_{ij} - s_{aj} s_{ik}}{\sqrt{4\, s_{ai} s_{ak}\, s_{ij} s_{jk}}}$$

Collinear factorization and resummation

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- Reference for $q \to q'$ upon integration over s_{ai}, x_a, ϕ_j given by NLO kernel

$$P_{qq'}(z) = C_F T_R \left[(1+z)\ln^2 z - \left(\frac{8}{3}z^2 + 9z + 5\right)\ln z + \frac{56}{9}z^2 + 4z - 8 - \frac{20}{9z} \right]$$

- So far we only have

$$P_{qq'}(z) = -C_F T_R \Big[5(1-z) + 2(1+z)\ln z \Big]$$

- Remainder scheme-dependent, must be computed in D dimensions
- Key is to realize that we just set up a local, modified subtraction method

$$\boldsymbol{P_{qq'}(z) = \left(\mathbf{I} + \frac{1}{\varepsilon}\mathcal{P} - \mathcal{I}\right)_{qq'}(z) + \int \mathrm{d}\Phi_{+1}(\mathbf{R}-\mathbf{S})_{qq'}(z,\Phi_{+1})}$$

where

$$\mathrm{I}_{qq'}(z) = \int \mathrm{d}\Phi_{+1} S_{qq'}(z,\Phi_{+1})\,, \qquad \mathcal{P}_{qq'}(z) = \int_z \frac{\mathrm{d}x}{x} P^{(0)}_{qg}(x) P^{(0)}_{gq}(z/x)$$

$$\mathcal{I}_{qq'}(z) = 2\int_z \frac{\mathrm{d}x}{x}\, C_F \left(\frac{1+(1-x)^2}{x}\ln(x(1-x)) + x \right) P_{gq}(z/x)$$

Collinear factorization and resummation

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- Analytical computation of I not needed, as $\mathrm{I}+\mathcal{P}/\varepsilon$ finite
- Simulate as endpoint, starting from $\mathcal{O}(\varepsilon)$ coefficient of integrand
  - Generate point in triple collinear phase space, but retroactively project onto $s_{ai}=0$
  - Guarantees phase-space coverage identical to fully differential simulation
- Kernel for endpoint contribution defined by $\Delta\mathrm{I}_{qq'}=\tilde{\mathrm{I}}_{qq'}-\tilde{\mathcal{I}}_{qq'}$, where

$$\tilde{\mathrm{I}}_{qq'}=C_FT_R\left[\frac{1+z_j^2}{1-z_j}+\left(1-\frac{2\,z_az_i}{(z_a+z_i)^2}\right)\left(1-z_j+\frac{1+z_j^2}{1-z_j}\right)\Big(\ln(z_az_iz_j)-1\Big)\right]$$

$$\tilde{\mathcal{I}}_{qq'}=2C_F\left[\frac{1+z_j^2}{1-z_j}\ln((z_a+z_i)\,z_j)+(1-z_j)\right]P_{gq}^{(0)}\Big(\frac{z_a}{z_a+z_i}\Big)\,.$$

- Cross-checked method analytically using phase space from
  [Gehrmann,Gehrmann-DeRidder,Heinrich] hep-ph/0311276 (timelike)
  [Ellis,Vogelsang] hep-ph/9602356 (spacelike)

Numerical implementation in Dire

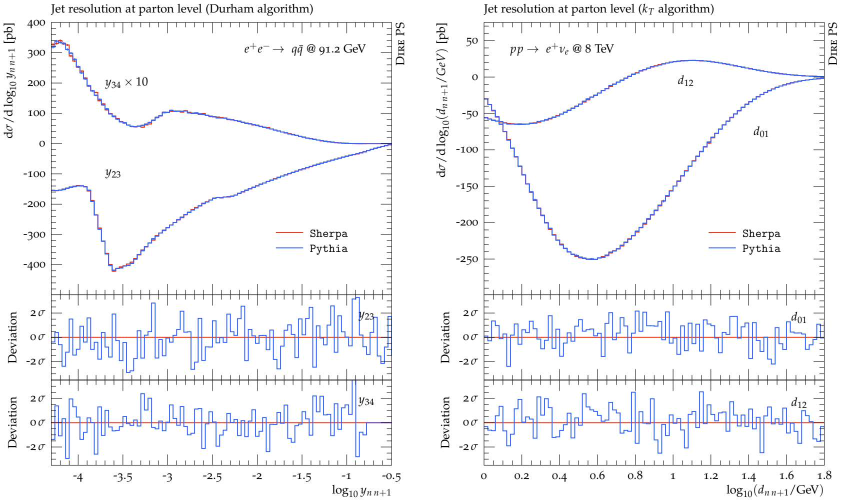
Basic layout of Dire [Prestel,SH] arXiv:1506.05057

- Dipole-like parton shower, kernels as close as possible to DGLAP
- Partial fraction soft eikonal à la Catani-Seymour, evolve in dipole-k_T
- Two independent implementations (Pythia & Sherpa)
- Cross-validation at particle level

New developments

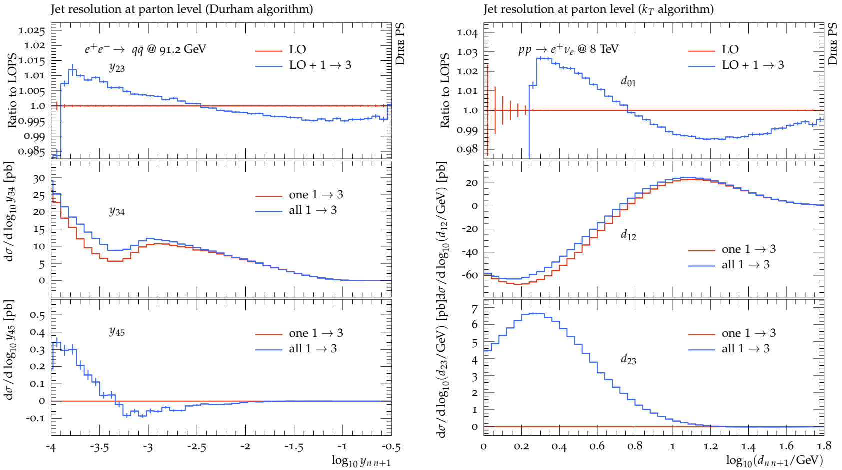
- MC counterterms implemented in Amegic & Comix [SH]
- MC@NLO matching & NLO subtraction in Sherpa [SH]
- UNLOPS / MEPS@NLO merging in Pythia / Sherpa [Prestel,SH]
- Flavor-changing triple collinear splitting functions [Prestel,SH] arXiv:1507.00742
- NLO DGLAP kernels & 3-loop cusp [Krauss,Prestel,SH] arXiv:1507.00982

Validation



- Effect of single $1 \to 3$ emission on leading and next-to-leading jet rate

Impact on leading-order PS prediction



- Effect of $1 \to 3$ emissions on leading jet rate
- Impact of multiple $1 \to 3$ emissions

Dire outlook

So far

- Developed MC algorithm to implement $2 \to 4$ splittings that recovers integrated NLO splitting functions for $q \to q'$ / $q \to \bar{q}$
- Cross-validated implementation Pythia $\leftrightarrow$ Sherpa

To-do list

- Extension to non flavor changing splitting kernels
- Treatment of color correlations in soft-gluon emissions

Thank you for your attention

Negative branching "probabilities" in Markov Chains

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- Problem in NLO splitting kernels, sub-leading color terms, etc. lies in negative weights → no-emission probability *locally* exceeds unity
- Recall standard veto algorithm: $\mathcal{P}_{\rm no}(t,t') = \exp\{F(t) - F(t')\}$
  Exact MC solution $t = F^{-1}\big[\,F(t') + \ln R\,\big]$, R – random number
- Don't want or can't compute $F(t) = -\int_t {\rm d}\bar{t} f(\bar{t})$,
  instead find simple function $g(t) > f(t)$ with integral $G(t)$
- Generate points according to $g(t)$ and accept with $f(t)/g(t)$

Standard probability for one acceptance with n rejections

$$\frac{f(t)}{g(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} {\rm d}\bar{t}\, g(\bar{t})\right\} \prod_{i=1}^{n} \left[\int_{t_{i-1}}^{t'} {\rm d}t_i \left(1 - \frac{f(t_i)}{g(t_i)}\right) g(t_i)\, \exp\left\{-\int_{t_i}^{t_{i+1}} {\rm d}\bar{t}\, g(\bar{t})\right\}\right]$$

Split weight into MC and analytic part using auxiliary function $h(t)$

$$\frac{f(t)}{h(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} {\rm d}\bar{t}\, g(\bar{t})\right\} \prod_{i=1}^{n} \left[\int_{t_{i-1}}^{t'} {\rm d}t_i \left(1 - \frac{f(t_i)}{h(t_i)}\right) g(t_i)\, \exp\left\{-\int_{t_i}^{t_{i+1}} {\rm d}\bar{t}\, g(\bar{t})\right\}\right]$$

$$w(t, t_1, \ldots, t_n) = \frac{h(t)}{g(t)} \prod_{i=1}^{n} \frac{h(t_i)}{g(t_i)} \frac{g(t_i) - f(t_i)}{h(t_i) - f(t_i)}$$

Negative branching “probabilities” in Markov Chains

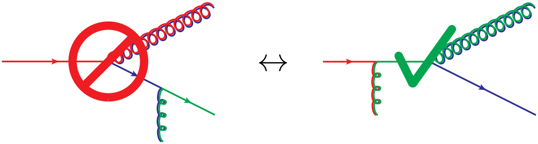
Looks trivial, surprisingly it’s not: It allows to

- Resum sub-leading color terms in MC@NLO and POWHEG
  [Krauss,Schönherr,Siegert,SH] arXiv:1111.1220
- Implement triple-collinear splitting functions in parton showers
  [Prestel,SH] arXiv:1705.00742
- Use PDFs with negative values in parton showers
  [Prestel,SH] arXiv:1506.05057
- Enhance branching probabilities in parton showers
  [Schumann,Siegert,SH] arXiv:0912.3501, [Lönnblad] arXiv:1211.7204
- Reweight parton showers [Bellm,Plätzer,Richardson,Siódmok,Webster] arXiv:1605.08256
  [Mrenna,Skands] arXiv:1605.08352, [Bothmann,Schönherr,Schumann] arXiv:1606.08753

Color coherence and the dipole picture

[Marchesini,Webber] NPB310(1988)461

- Individual color charges inside a color dipole cannot be resolved by gluons of wavelength larger than the dipole size
  $\rightarrow$ emission off combined mother parton instead

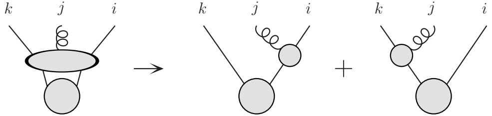


- Net effect is destructive interference outside cone with opening angle defined by emitting color dipole
  $\rightarrow$ Soft anomalous dimension halved due to reduced phase space
- Formerly implemented by angular ordering / angular veto
  [Webber at al.] hep-ph/0210213, [Sjöstrand et al.] hep-ph/0603175
- Alternative description in terms of color dipoles
  [Gustafsson,Pettersson] NPB306(1988)746, [Kharraziha,Lönnblad] hep-ph/9709424
  [Winter,Krauss] arXiv:0712.3913

The midpoint between dipole and parton showers

- Angular ordered / vetoed parton shower does not fill full phase space
  Dipole shower lacks parton interpretation → prefer alternative to both
- Can preserve parton picture by partial fractioning soft eikonal
  ↔ soft enhanced part of splitting function [Catani,Seymour] hep-ph/9605323

$$\frac{p_i p_k}{(p_i p_j)(p_j p_k)} \to \frac{1}{p_i p_j}\frac{p_i p_k}{(p_i + p_k)p_j} + \frac{1}{p_k p_j}\frac{p_i p_k}{(p_i + p_k)p_j}$$



- "Spectator"-dependent kernels, singular in soft-collinear region only
  → capture dominant coherence effects (3-parton correlations)

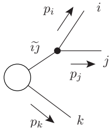
$$\frac{1}{1-z} \to \frac{1-z}{(1-z)^2 + \kappa^2} \qquad \kappa^2 = \frac{k_\perp^2}{Q^2}$$

- For correct soft evolution, ordering variable must be identical
  at both "dipole ends" (→ recover soft eikonal at integrand level)

The midpoint between dipole and parton showers

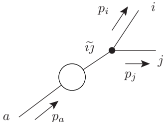
Choose parametrization such that soft term is $\frac{1-z}{(1-z)^2+\kappa^2}$ in all dipole types

(1) FF



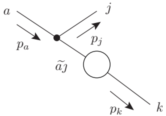
$$\kappa^2 = \frac{p_i p_j \, p_j p_k}{(p_{\widetilde{ij}} p_{\tilde{k}})^2} \qquad z_j = \frac{p_j p_k}{p_{\widetilde{ij}} p_{\tilde{k}}}$$

(2) FI



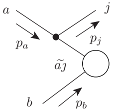
$$\kappa^2 = \frac{p_i p_j \, p_j p_a}{(p_{ij} p_a)^2} \qquad z_j = \frac{p_j p_a}{p_{ij} p_a}$$

(3) IF



$$\kappa^2 = \frac{p_a p_j \, p_j p_k}{(p_{jk} p_a)^2} \qquad z_j = \frac{p_j p_k}{p_{jk} p_a}$$

(4) II



$$\kappa^2 = \frac{p_a p_j \, p_j p_b}{(p_a p_b)^2} \qquad z_j = \frac{p_j p_b}{p_a p_b}$$

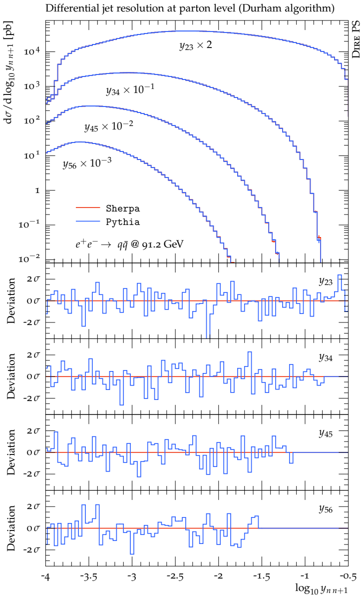
Preserve collinear anomalous dimensions & sum rules → splitting functions fixed

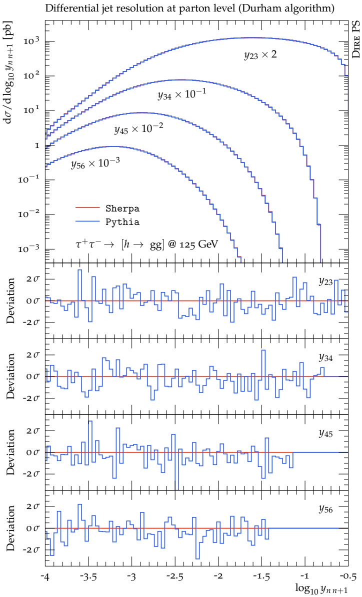
$$P_{qq}(z,\kappa^2) = 2\,C_F\left[\left(\frac{1-z}{(1-z)^2+\kappa^2}\right)_+ - \frac{1+z}{2}\right] + \gamma_q\,\delta(1-z)$$

$$P_{gg}(z,\kappa^2) = 2\,C_A\left[\left(\frac{1-z}{(1-z)^2+\kappa^2}\right)_+ + \frac{z}{z^2+\kappa^2} - 2 + z(1-z)\right] + \gamma_g\,\delta(1-z)$$

$$P_{qg}(z,\kappa^2) = 2\,C_F\left[\frac{z}{z^2+\kappa^2} - \frac{2-z}{2}\right] \qquad P_{gq}(z,\kappa^2) = T_R\left[z^2+(1-z)^2\right]$$

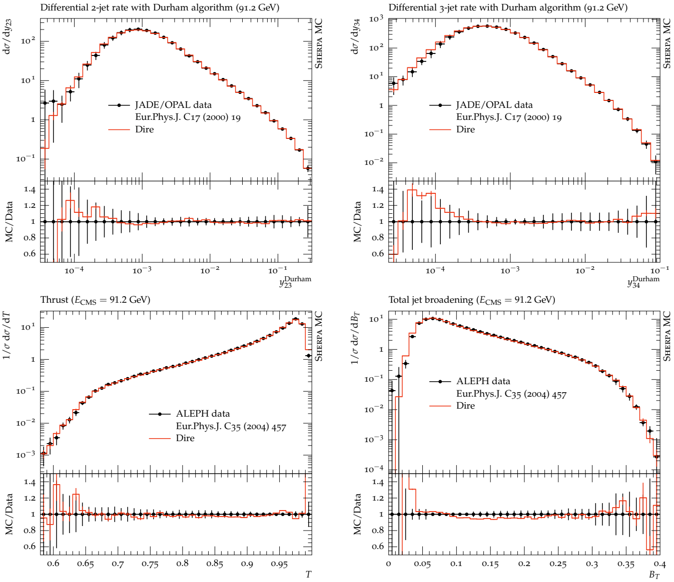
Validation in $e^+e^- \to$ hadrons





Comparison to LEP data

[Prestel,SH] arXiv:1506.05057



Dire as an NLO subtraction method and MC@NLO

[SH] TBP?

- Can view new shower model as modification of CS subtraction
- IR counterterms computed and implemented in Sherpa (improved cancellation in $pp \to h + j$ due to regulated $1/z$ terms)

- Sherpa MC@NLO based on exponentiation of CS dipole subtraction terms

  [Krauss,Siegert,Schönherr,SH]
  arXiv:1111.1220, arXiv:1208.2815

- Dire modified CS subtraction automatically available for MC@NLO matching
- Interesting differences due to evolution variables and kernels